NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

ERROR PROBABILITIES OF FFH/BFSK WITH
NOISE NORMALIZATION AND SOFT DECISION
VITERBI DECODING IN A FADING CHANNEL
WITH PARTIAL-BAND JAMMING

by

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March 1994

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94 6 7 108

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE March 1994	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE ERROR PROBABILITIES OF FFH/BFSK WITH NOISE NORMALIZATION AND SOFT DECISION VITERBI DECODING IN A FADING CHANNEL WITH PARTIAL-BAND JAMMING	5. FUNDING NUMBERS
6. AUTHOR(S) CHUA, ANTHONY, Y. P.	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey CA 93943-5000	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	12b. DISTRIBUTION CODE A

13. ABSTRACT *(maximum 200 words)*

An error probability analysis of a communications link employing convolutional coding with soft decision Viterbi decoding implemented on a fast frequency-hopped, binary frequency-shift keying (FFH/BFSK) spread spectrum system is performed. The signal is transmitted through a frequency non-selective, slowly fading channel with partial-band jamming. Noise normalization combining is employed at the receiver to alleviate the effects of partial-band jamming. The received signal amplitude of each hop is modeled as a Rician process, and each hop is assumed to fade independently.

It is found that with the implementation of soft decision Viterbi decoding that the performance of the receiver is improved dramatically when the coded bit energy to partial-band noise power spectral density ratio (E_b/N_I) is greater than 10dB. At higher E_b/N_I, the asymptotic error improves dramatically and varies from 10^{-6} to 10^{-12} depending on the constraint length (ν), number of hops/bit (L), and the strength of the direct signal $(\alpha^2/2\sigma^2)$. In addition, nearly worst case jamming occurs when the jammer uses a full band jamming strategy, even when L=1 and there is a very strong direct signal $(\alpha^2/2\sigma^2 = 100)$. Due to non-coherent combining losses, when the hop per bit ratio is increased, there is some degradation at moderate E_b/N_I. Furthermore, when a stronger code is used (i.e., the constraint length is longer), performance improves, especially for high E_b/N_I where the asymptotic error is reduced. Finally, soft decision decoding improves performance over hard decision decoding from 4 to 8dB at moderate E_b/N_I depending on the code rate and with a much lower asymptotic error at high E_b/N_I.

14. SUBJECT TERMS fast frequency hopping, noise normalization, convolutional coding	15. NUMBER OF PAGES 75
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL

Error Probabilities of FFH/BFSK with Noise Normalization
and Soft Decision Viterbi Decoding in a Fading Channel
with Partial-Band Jamming

by

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Ministry of Defense, Singapore
B.S.E.E., National University of Singapore, 1986

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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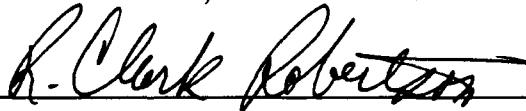NAVAL POSTGRADUATE SCHOOL
March 1994
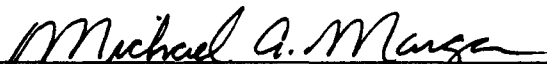
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ABSTRACT

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LIST OF SYMBOLS

α^2	Average Power of the Non-faded (direct) component of the signal
$2\sigma^2$	Average Power of the Rayleigh-faded (diffuse) component of signal
σ_k^2	Average Power of the Noise at hop k
γ	Fraction of hops jammed
λ	$(2\sigma^2 + \sigma_k^2) \, / \, (2\sigma_k^2 \, \sigma^2)$
ψ	$\sqrt{\{(2z_{1k}) \, / \, \sigma_k\}}$
ς	$\alpha^2 \, / \, \sigma^2$
ξ_k	$2\sigma^2 \, / \, \sigma_k^2$
ρ_k	$\alpha^2 \, / \, \sigma_k^2$
β_k	$1 \, / \, (2(1 + \xi_k))$
ω_j	Number of paths for event of weight j
A_i^n	$n(n-1)(n-2)\ldots\ldots(n-i+1)$
a_k	RMS amplitude of signal
B	Bandwidth of the signal
d_{free}	Hamming free distance
E_b	Uncoded bit energy
E_c	Coded bit energy
N_I	Power Spectral Density of the Jamming Noise
N_0	Power Spectral Density of the Thermal Noise
N_T	Power Spectral Density of the Total Noise
P_b	Probability of Bit error

P_j Probability of an error event of weight j

$P_j(\underline{i})$ Conditional probability of an error event given that $\underline{i}$ hops are jammed

P_T Total Noise Power

R_b Information Bit Rate

R_h Hop Rate

X_{1k}, X_{2k} Received Signal at Branch 1 and 2

Z_{1k}, Z_{2k} Normalized received signal at Branch 1 and 2

I. INTRODUCTION

An important characteristic of military communications is the requirement to be robust against hostile interference or other unintentional interference. Fast frequency-hopping spread spectrum systems are widely used in military communications to mitigate the effects of hostile interference. Fast frequency-hopping implies that each bit is transmitted multiple times at different carrier frequencies determined by some pseudorandom sequence known only by the transmitter and intended receiver. As a result, the hostile interference must be spread over a much wider bandwidth which results in a lower interference noise power spectral density.

Fast frequency-hopped systems are susceptible to partial-band interference when the demodulator assigns equal weight to each of the multiple transmissions, or hops. As a result, various methods of signal normalization to overcome the adverse effects of partial-band jamming have been considered. Previous works have studied the performance of receivers utilizing noise normalization [Ref. 1], ratio-statistic normalization [Ref. 2], and self-normalization [Ref. 3].

For receivers utilizing noise normalization, convolutional coding with hard decision decoding [Ref. 1] has been studied. It was found that with hard decision decoding that performance is improved significantly as compared to the uncoded system. The implementation of soft decision decoding is expected to further improve performance [Ref. 4]. The analysis and actual implementation of a receiver utilizing soft decision

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decoding requires much more computing power than a receiver with either no coding or one with hard decision decoding since the decision on the coded transmissions are not made until they are in the Viterbi decoder. With improved computing capability, soft decision convolutional coding is now practical and will be implemented in the new generation of frequency-hopping radios. The investigation of the performance of systems using convolutional coding and soft decision Viterbi decoding is an important problem as such systems will be installed in satellite and other military systems.

An error probability analysis of a communications link employing convolutional coding with soft decision Viterbi decoding of a fast frequency-hopped, binary frequency-shift keying (FFH/BFSK) spread spectrum system with non-coherent, noise-normalized detection and a channel with Rician fading and partial-band jamming is performed in this thesis. This is a continuation of the research conducted in [Ref. 1] with the implementation of convolutional coding with soft decision Viterbi decoding.

The data are first encoded by convolutional coding. After that, the transmitter transmits L hops for each encoded bit. The channel is modeled as a Rician fading channel. Both partial- band interference and thermal noise are also assumed to affect the channel. The partial-band interference is assumed to be unaffected by the fading channel.

The FFH/BFSK receiver with noise normalization and soft decision Viterbi decoding is shown in Fig. 1. At the receiver, perfect dehopping is assumed and the signals are demodulated by a pair of quadrature receivers. Before going to the soft-decision Viterbi decoder, the signals are passed through a noise normalization combiner.

At the noise normalization combiner, the noise power, estimated using a noise-only channel estimator, is inverted and used to multiply each hop of an encoded bit.

Each hop is assumed to fade independently, and the channel for each hop is modeled as a frequency non-selective, slowly fading Rician process. Given these assumptions, the implication is that the channel's coherence bandwidth is smaller than the smallest frequency hop spacing but larger than the dehopped signal bandwidth, and the channel coherence time is larger than the hop duration. The amplitude of the dehopped signal is modeled as a Rician random variable and consists of a non-faded (direct) component and a Rayleigh-faded (diffuse) component [Ref. 4].

There are two types of interference considered in this study. The first is partial-band noise jamming which can be caused either by a partial-band jammer or some unintended narrowband interference. If the fraction of the spread bandwidth jammed is γ, then the partial-band noise jamming, modeled as additive Gaussian noise and assumed to be present in both branches of the BFSK demodulator when present, affects each bit with probability γ. If the average power spectral density of the interference over the entire spread bandwidth is $N_I/2$, then the power spectral density of the partial-band interference is $N_I/(2\gamma)$ when it is present. The second type of interference considered is wideband interference and is assumed to be caused by thermal noise or other wideband interferences. The power spectral density of this wideband noise is defined as $N_O/2$. Of course, when $\gamma = 1$, the partial-band interference is also essentially wideband in nature from the perspective of the receiver.

The power spectral density of the total noise, $N_T/2$, is

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$$\frac{N_T}{2} = \begin{cases} \dfrac{N_I}{2\gamma} + \dfrac{N_O}{2} & \text{when jamming is present} \\[2ex] \dfrac{N_O}{2} & \text{when jamming is absent} \end{cases} \qquad (1)$$

If the noise normalized BFSK demodulator has a noise equivalent bandwidth of B Hz, then the received noise power for a given hop k is

$$\sigma_k^2 = \begin{cases} \left(\dfrac{N_I}{\gamma} + N_O\right) B & \text{with probability } \gamma \\[2ex] N_O B & \text{with probability } (1-\gamma) \end{cases} \qquad (2)$$

The bit rate is assumed to be R_b. With L hops/bit, the hop rate $R_h = LR_b$. Therefore, the noise equivalent bandwidth, B, must satisfy $B \geq R_h$. For this analysis, B is assumed to be equal to R_h. However, the overall spread bandwidth is assumed to be very large as compared to the hop rate. As the bit energy is assumed to be constant, the coded bit energy, E_c, is related to the uncoded bit energy, E_b, and the code rate, r, by $E_c = r E_b$.

It is well known that in order to maintain the same data bit rate for a coded system as for the uncoded system that the transmission bandwidth must be increased by $1/r$. For conventional system this may be a disadvantage, but for spread spectrum systems where the intent is to maximize transmission bandwidth, this increase is advantageous.

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II. ANALYSIS

In this thesis, an analysis of the bit error probability for the receiver illustrated in Fig. 1 is performed. This analysis requires the statistics of the sampled outputs, x_{ik}, $i = 1,2$ of the quadrature receiver for each branch and the noise power, σ_k^2 for each hop k of a bit.

A. PROBABILITY OF BIT ERROR, P_b

It is well known that for soft-decision Viterbi decoding that the probability of bit error is upper bounded by [Ref. 4-6]

$$P_b \leq \frac{1}{k} \sum_{j=d_{free}}^{\infty} \tilde{\omega}_j \, P_j \tag{3}$$

where   k is the number of input bits to the encoder,

         $\tilde{\omega}_j$ is the total information weight of a path of weight j [Ref. 5],

         P_j is the probability that the all-zero path is eliminated by a path of weight

            j, and

         d_{free} is the free distance of the code.

This is the standard union bound used to evaluate the performance of coded systems. The tightness of the bound can only be established by comparison with a simulation of the system under consideration since an analytic solution does not exist. The bound is

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reasonably tight for a conventional MFSK system [Ref. 5]. Consequently, it is expected that the bound will also be tight for the noise normalized receiver, especially when the number of hops per bit is small.

Since all the hops are independent and identically distributed, P_j is derived from

$$P_j = \left(\sum_{i_1=0}^{L} \binom{L}{i_1} \gamma^{i_1}(1-\gamma)^{L-i_1} p_1(i_1) \right) \left(\sum_{i_2=0}^{L} \binom{L}{i_2} \gamma^{i_2}(1-\gamma)^{L-i_2} p_2(i_2) \right) \cdots$$
$$\left(\sum_{i_j=0}^{L} \binom{L}{i_j} \binom{L}{i_j} \gamma^{i_j} (1-\gamma)^{L-i_j} p_j(i_j) \right) \tag{4}$$

where $p_p(i_p)$ is probability of error given that i_p of L hops in p^{th} coded transmission are jammed. This can be simplified to

$$P_j = \sum_{i_j=0}^{L} \sum_{i_{j-1}=0}^{L} \cdots \sum_{i_1=0}^{L} \binom{L}{i_1} \binom{L}{i_2} \cdots \binom{L}{i_j}$$
$$\gamma^{\sum_{l=1}^{j} i_l} (1-\gamma)^{jL - \sum_{l=1}^{j} i_l} P_j(\underline{i}) \tag{5}$$

where $\underline{i} = [i_1 \ i_2 \ \ldots \ i_j]$, and $P_j(\underline{i})$ is the conditional probability tha the all-zero path is eliminated by a path of weight j given that $\underline{i}$ hops are jammed and equals $(p_1(i_1)\ldots p_j(i_j))$.

Finally, it is possible to simplify (5) to a simple binomial expression cf the form

$$P_j = \sum_{i_j=0}^{jL} \binom{jL}{\sum_{p=1}^{j} i_p} \gamma^{\sum_{p=1}^{j} i_p} (1-\gamma)^{jL - \sum_{p=1}^{j} i_p} P_j(\underline{i}) \tag{6}$$

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From [Ref. 6 : pp 34], the conditional probability $P_j(i)$ can be calculated for binary orthogonal signaling and noncoherent detection and is equal to the probability that the sum of j samples from the signal-absent quadratic detector is greater than the sum of j samples from the signal present detector. Hence,

$$P_j(i) = 1 - \int_0^\infty f_{Z_1}(z_1|i) \left[\int_0^{z_1} f_{Z_2}(z_2) \, dz_2 \right] dz_1 \qquad (7)$$

where Z_1 and Z_2 are the random variable of the noise normalized outputs of the branches 1 and 2 respectively and is summed over j coded transmissions. It is assumed, without loss of generality, that the signal is present in branch 1.

From (7), it is observed that in order to determine the bound on bit error probabilities given by (3) for a noise normalized receiver and a Rician fading channel, we need to first determine the conditional probability density function of Z_1 and the probability density function of Z_2, the noise normalized decision statistics which are formed by summing the noise normalized, sampled outputs for each hop over j coded transmissions [Ref. 4,6]. This summation of the j coded transmissions is made inside the Viterbi decoder. It should be noted, with reference to Fig. 1, that no decision is made until the differential amplifier output passes through the Viterbi decoder.

B. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLE Z_1

1. Probability Density Function of Z_{1k}

To determine the probability density function of the noise normalized, sampled outputs for each hop, we let σ_k^2 represent the noise power in a given hop k of a bit. For this thesis, we assume that σ_k^2 is determined from the noise-only channel estimator and is known exactly. This results in an ideal performance analysis. Furthermore, without loss of generality, the signal is assumed to be in branch 1 of the BFSK demodulator. To determine the probability density function of the noise normalized random variable Z_{1k}, we start with the conditional probability density function of the random variable X_{1k} at the output of the quadratic detector. This output is equivalent to the envelope squared of a sine wave plus a narrowband process. The probability density function of the random variable used to model this signal is [Ref. 7 : pp 112, eq. 4-69]

$$p(q \mid \theta) = \frac{1}{2\sigma^2} \exp\left[-\frac{(q+A^2)}{2\sigma^2}\right] I_0\left[\frac{Aq^{1/2}}{\sigma^2}\right]$$

$$\text{for } q > 0 \tag{8}$$

where   A = rms signal amplitude,

σ^2 = variance of the narrowband process, and

$I_0(\bullet)$ = modified Bessel function of the first kind and order zero.

If σ_k^2 is the average noise power in a given hop k and the signal amplitude is $a_k\sqrt{2}$, the resultant conditional probability density function is

$$f_{X_{1k}}(x_{1k} \mid a_k) = \frac{1}{2\sigma_k^2} \exp\left[\frac{-(x_{1k}+2a_k^2)}{2\sigma_k^2}\right] I_0\left(\frac{a_k\sqrt{2x_{1k}}}{\sigma_k^2}\right) u(x_{1k}) \qquad (9)$$

where $u(\bullet)$ is the unit step function.

Next, since the signal amplitude for each hop is modeled as a Rician random variable, then [Ref. 8 : pp. 592, eq. 7-53]

$$f_{A_k}(a_k) = \frac{a_k}{\sigma^2} \exp\left[-\frac{a_k+\alpha^2}{2\sigma^2}\right] I_0\left[\frac{a_k\alpha}{\sigma^2}\right] u(a_k) \qquad (10)$$

where the total average signal power is a_k^2, which is a summation of α^2 {average power of the non-faded (direct) component} and $2\sigma^2$ {average power of the Rayleigh faded (diffuse) component}. The total average signal power is assumed to remain constant from hop to hop. It should be noted that when $\alpha^2 = 0$, the channel is a Rayleigh fading model, and if $2\sigma^2 = 0$, there is no fading.

Furthermore, the noise normalized received signal is related to the sampled outputs by

$$Z_{1k} = \frac{X_{1k}}{\sigma_k^2} \qquad (11)$$

In order to determine the conditional probability density function of the noise normalized random variable, we perform the linear transformation (11) to obtain

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$$f_{Z_{1k}}(z_{1k} \mid a_k) = f_{X_{1k}}(x_{1k} = z_{1k} \mid a_k) \frac{dx_{1k}}{dz_{1k}}\bigg|_{x_{1k}=z_{1k}} \tag{12}$$

Substituting (9) into (12), we get the conditional probability density function of Z_{1k}

$$f_{Z_{1k}}(z_{1k} \mid a_k) = \frac{1}{2} \exp\left[-\frac{\sigma_k^2 z_{1k} + 2a_k^2}{2\sigma_k^2}\right] I_0\left[\frac{a_k\sqrt{2z_{1k}}}{\sigma_k}\right] u(z_{1k}) \tag{13}$$

The unconditional probability density function of Z_{1k} is now found from

$$f_{Z_{1k}}(z_{1k}) = \int_0^\infty f_{Z_{1k}}(z_{1k} \mid a_k) f_{A_k}(a_k) \, da_k \tag{14}$$

Substituting (10) and (13) into (14), we get

$$
\begin{aligned}
f_{Z_{1k}}(z_{1k}) &= \int_0^\infty f_{Z_{1k}}(z_{1k} \mid a_k) f_{A_k}(a_k) \, da_k \\
&= \int_0^\infty \frac{1}{2} \exp\left[-\frac{\sigma_k^2 z_{1k} + 2a_k^2}{2\sigma_k^2}\right] I_0\left[\frac{a_k\sqrt{2z_{1k}}}{\sigma_k}\right] \\
&\qquad\qquad \frac{a_k}{\sigma_k} \exp\left[-\frac{a_k^2 + \alpha^2}{2\sigma^2}\right] I_0\left[\frac{a_k\alpha}{\sigma^2}\right] da_k
\end{aligned}
\tag{15}
$$

which can be evaluated to yield

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$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2\sigma^2} \exp\left[-\left(\frac{z_{1k}}{2}+\frac{\alpha^2}{2\sigma^2}\right)\right]\int_0^\infty a_k \exp\left[-a_k^2\left(\frac{1}{\sigma_k^2}+\frac{1}{2\sigma^2}\right)\right]$$
$$I_0\left[a_k\frac{\sqrt{2z_{1k}}}{\sigma_k}\right]I_0\left[\frac{a_k\alpha}{\sigma^2}\right]\,da_k$$

$$= \frac{1}{2\sigma^2}\exp\left[-\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2}\right]\int_0^\infty a_k \exp\left[-a_k^2\left(\frac{2\sigma^2+\sigma_k^2}{2\sigma_k^2\sigma^2}\right)\right]$$
$$I_0\left[a_k\frac{\sqrt{2z_{1k}}}{\sigma_k}\right]I_0\left[\frac{a_k\alpha}{\sigma^2}\right]\,da_k$$

(16)

Here, we make the substitutions

$$\lambda = \frac{2\sigma^2+\sigma_k^2}{2\sigma_k^2\sigma^2}$$

$$\psi = \frac{\sqrt{2z_{1k}}}{\sigma_k}$$

(17)

$$\varsigma = \frac{\alpha}{\sigma^2}$$

to get

$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2\sigma^2}\exp\left[-\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2}\right]\int_0^\infty a_k \exp\left[-\lambda a_k^2\right]$$
$$I_0[\psi a_k]\,I_0[\varsigma a_k]\,da_k$$

(18)

From [Ref. 9 : pp 238, eq 4.436],

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$$\int_0^\infty e^{-\rho^2 x^2} J_p(ax) J_p(bx) x \ dx = \frac{1}{2\rho^2} \exp\left[-\frac{a^2+b^2}{4\rho^2}\right] I_p\left[\frac{ab}{2\rho^2}\right] \qquad (19)$$

If we let $p = 0$ and since $I_0(x) = J_0(ix)$, (19) becomes

$$\int_0^\infty e^{-\rho^2 x^2} I_0(iax) I_0(ibx) x \ dx = \frac{1}{2\rho^2} \exp(-\frac{a^2+b^2}{4\rho^2}) \qquad (20)$$
$$I_0(\frac{ab}{2\rho^2})$$

If we let $\psi = ia$, $\varsigma = ib$, $\lambda = \rho^2$, and $a_k = x$, the integral in (18) becomes

$$\int_0^\infty a_k \exp\left[-\lambda a_k^2\right] I_0\left[\psi a_k\right] I_0\left[\varsigma a_k\right] \ da_k$$

$$= \frac{1}{2\lambda} \exp\left[-\frac{(i\psi)^2+(i\varsigma)^2}{4\lambda}\right] I_0\left[\frac{(i\psi)(i\varsigma)}{2\lambda}\right]$$

$$= \frac{1}{2\lambda} \exp\left[\frac{\psi^2+\varsigma^2}{4\lambda}\right] I_0\left[-\frac{\psi\gamma}{2\lambda}\right] \qquad (21)$$

$$= \frac{1}{2\lambda} \exp\left[\frac{\psi^2+\varsigma^2}{4\lambda}\right] I_0\left[\frac{\psi\gamma}{2\lambda}\right]$$

$$\text{since } I_0(-x) = I_0(x)$$

Substituting this into (18), we get

$$f_{z_{1k}}(z_{1k}) = \frac{1}{2\sigma^2} \exp\left[-\frac{z_{1k}\sigma^2+\sigma^2}{2\sigma^2}\right] \frac{1}{2\lambda} \exp\left[\frac{\psi^2+\varsigma^2}{4\lambda}\right] I_0\left[\frac{\psi\gamma}{2\lambda}\right] \qquad (22)$$

Now substituting in (17), we get

$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2\sigma^2} \frac{2\sigma_k^2\sigma^2}{2(2\sigma^2+2\sigma_k^2)}$$

$$\exp\left[-\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2} : \frac{\frac{2z_{1k}}{\sigma_k^2}+\frac{\alpha^2}{\sigma^4}}{4\left(\frac{2\sigma^2+\sigma_k^2}{2\sigma_k^2\sigma^2}\right)}\right] I_0\left[\frac{\frac{\sqrt{2z_{1k}}}{\sigma_k}\frac{\alpha}{\sigma^2}}{2\left(\frac{2\sigma^2+\sigma_k^2}{2\sigma_k^2\sigma^2}\right)}\right] \quad (23)$$

This can be simplified to

$$f_{Z_{1k}}(z_{1k}) = \frac{\sigma_k^2}{2\sigma^2+\sigma_k^2}\exp\left[-\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2}+\frac{(2z_{1k}\sigma^4+\alpha^2\sigma_k^2)\,(\sigma_k^2\sigma^2)}{(\sigma_k^2\sigma^4)[\,2(2\sigma^2+\sigma_k^2)]}\right]$$

$$I_0\left[\frac{\sqrt{2z_{1k}}\alpha}{2\sigma^2+\sigma_k^2}\right]$$

$$= \frac{1}{2\left(\frac{2\sigma^2}{\sigma_k^2}+1\right)}\exp\left[-\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2}+\frac{2z_{1k}\sigma^4+\alpha^2\sigma_k^2}{2\sigma^2\sigma_k^2\left(\frac{2\sigma^2}{\sigma_k^2}+1\right)}\right] \quad (24)$$

$$I_0\left[\frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{\left(\frac{2\sigma^2}{\sigma_k^2}+1\right)}\right]$$

Next, we substitute the signal-to-noise ratio of the diffuse component, $\xi_k = 2\sigma^2/\sigma_k^2$, to

get

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$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2(1+\xi_k)} I_0\left[\frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{(1+\xi_k)}\right] \exp\left[-\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2} + \frac{2z_{1k}\sigma^4+\alpha^2\sigma_k^2}{2\sigma^2\sigma_k^2(1+\xi_k)}\right]$$

$$= \frac{1}{2(1+\xi_k)} I_0\left[\frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{(1+\xi_k)}\right] \qquad (25)$$

$$\exp\left[\frac{-(z_{1k}\sigma^2+\alpha^2)(2\sigma^2+\sigma_k^2)+2z_{1k}\sigma^4+\alpha^2\sigma_k^2}{2\sigma^2\sigma_k^2(1+\xi_k)}\right]$$

which can be simplified to

$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2(1+\xi_k)} \exp\left[-\frac{z_{1k}\sigma^2\sigma_k^2+2\alpha^2\sigma^2}{2\sigma^2\sigma_k^2(1+\xi_k)}\right] I_0\left[\frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{(1+\xi_k)}\right]$$

$$\qquad (26)$$

$$= \frac{1}{2(1+\xi_k)} \exp\left[-\frac{\frac{z_{1k}}{2}+\frac{\alpha^2}{\sigma_k^2}}{1+\xi_k}\right] I_0\left[\frac{\sqrt{2z_{1k}\frac{\alpha^2}{\sigma_k^2}}}{(1+\xi_k)}\right]$$

Finally, substituting the signal-to-noise ratio of the direct component, $\rho_k = \alpha^2 / \sigma_k^2$, we get

$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2(1+\xi_k)} \exp\left[-\frac{\frac{z_{1k}}{2}+\rho_k}{1+\xi_k}\right] I_0\left[\frac{\sqrt{2z_{1k}\rho_k}}{(1+\xi_k)}\right] \qquad (27)$$

Therefore, the probability density function of Z_{1k} has been determined.

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2. Probability Density Function of Z_1

Let $Z_{1kp}^{(1)}$ and $Z_{1kp}^{(2)}$ denote the random variable Z_{1kp} when hop is jammed or not jammed, respectively. For convolutional coding with soft decision Viterbi decoding, we need to determine the random variable for the sum of j coded transmissions [Ref. 9-10]. Assuming that for a coded transmission p of the j transmissions, i_p hops are jammed, we obtain the resultant decision random variable Z_1

$$Z_1 = \sum_{p=1}^{j} \sum_{k=1}^{L} Z_{1kp}$$

$$= \sum_{p=1}^{j} \sum_{k=1}^{i_p} Z_{1kp}^{(1)} + \sum_{p=1}^{j} \sum_{k=i_p+1}^{L} Z_{1kp}^{(2)}$$

(28)

where it can be seen that i_p is not necessarily the same for each of the j coded transmissions.

Since $Z_{1kp}^{(n)}$, $n=1,2$ are independent and identically distributed random variables,

$$f_{Z_{1k1}^{(n)}}(z_{1k1}^{(n)}) = f_{Z_{1k2}^{(n)}}(z_{1k2}^{(n)}) = \ldots\ldots = f_{Z_{1kj}^{(n)}}(z_{1kj}^{(n)}) \qquad n = 1,2 \qquad (29)$$

we can simplify (28) to

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$$Z_1 = \sum_{p=1}^{j} \sum_{k=1}^{L} Z_{1kp}$$

$$= \sum_{k=1}^{\sum_{p=1}^{j} i_p} Z_{1k}^{(1)} + \sum_{k=\left(\sum_{p=1}^{j} i_p\right)+1}^{jL} Z_{1k}^{(2)} \tag{30}$$

$$= \sum_{k=1}^{\sum_{p=1}^{j} i_p} Z_{1k}^{(1)} + \sum_{k=1}^{jL-\sum_{p=1}^{j} i_p} Z_{1k}^{(2)}$$

In order to determine the probability density function of Z_1, we need to find the Laplace transform of the probability density function of $Z_{1k}^{(n)}$. The probability density function is obtained from (27) by replacing ρ_k and ξ_k with $\rho_k^{(n)}$ and $\xi_k^{(n)}$, respectively, to get

$$f_{Z_{1k}^{(n)}}(z_{1k}^{(n)}) = \frac{1}{2(1+\xi_k^{(n)})} \exp\left[-\frac{\frac{z_{1k}^{(n)}}{2}+\rho_k^{(m)}}{1+\xi_k^{(n)}}\right] I_0\left[\frac{\sqrt{2z_{1k}^{(n)}\rho_k^{(n)}}}{(1+\xi_k^{(n)})}\right] \tag{31}$$

The Laplace transform is found from

$$F_{Z_{1k}^{(n)}}(s) = \int_0^\infty f_{Z_{1k}^{(n)}}(z_{1k}^{(n)}) \exp(-sz_{1k}^{(n)}) \, dz_{1k}^{(n)}$$

$$= \int_0^\infty \frac{1}{2(1+\xi_k^{(n)})} \exp\left[-\frac{1}{2}\left(\frac{z_{1k}^{(n)}+2\rho_k^{(n)}}{1+\xi_k^{(n)}}\right)\right] I_0\left(\frac{\sqrt{2\rho_k^{(n)}z_{1k}^{(n)}}}{1+\xi_k^{(n)}}\right) \tag{32}$$
$$\exp(-sz_{1k}^{(n)}) \, dz_{1k}^{(n)}$$

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This is simplified to

$$F_{Z_{1k}^{(n)}}(s) = \frac{1}{2(1+\xi_k^{(n)})} \exp\left(-\frac{\rho_k^{(n)}}{1+\xi_k^{(n)}}\right) \int_0^\infty \exp\left(-\frac{z_{1k}^{(n)}}{2(1+\xi_k^{(n)})} - s z_{1k}^{(n)}\right)$$
$$I_0\left(\frac{\sqrt{2\rho_k^{(n)} z_{1k}^{(n)}}}{1+\xi_k^{(n)}}\right) dz_{1k}^{(n)} \qquad (33)$$

If we substitute

$$\beta_k^{(n)} = \frac{1}{2(1+\xi_k^{(n)})} \qquad (34)$$

then

$$F_{Z_{1k}^{(n)}}(s) = \beta_k^{(n)} \exp\left(-2\rho_k^{(n)}\beta_k^{(n)}\right) \int_0^\infty \exp\left(-z_{1k}^{(n)}(\beta_k^{(n)}+s)\right)$$
$$I_0\left(2\beta_k^{(n)}\sqrt{2\rho_k^{(n)} z_{1k}^{(n)}}\right) dz_{1k}^{(n)} \qquad (35)$$

Next, we let $z_{1k}^{(n)} = u^2$ in (35) which implies

$$z_{1k}^{(n)} = 0 \quad \Rightarrow \quad u = 0$$
$$z_{1k}^{(n)} \to \infty \quad \Rightarrow \quad u \to \infty \qquad (36)$$

Making these substitutions in (35), we get

$$F_{Z_{1k}^{(n)}}(s) = 2\beta_k^{(n)} \exp\left(-2\rho_k^{(n)}\beta_k^{(n)}\right) \int_0^\infty \exp\left(-u^2(\beta_k^{(n)}+s)\right)$$
$$I_0\left(2\beta_k^{(n)} u\sqrt{2\rho_k^{(n)}}\right) u \, du \qquad (37)$$

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From [Ref. 10 : pp 486, eq 11.4.29],

$$\int\limits_{0}^{\infty} e^{-a^2 t^2} \, t^{\nu+1} \, J_\nu(bt) \, dt = \frac{b^\nu}{(2a^2)^{\nu+1}} \, e^{-b^2/4a^2} \qquad (38)$$

If we let $\nu = 0$, and since $I_0 = J_0(ix)$, (38) becomes

$$\int\limits_{0}^{\infty} e^{-a^2 t^2} \, t \, I_0(ibt) \, dt = \frac{1}{2a^2} \, e^{-b^2/4a^2} \qquad (39)$$

In addition, we let

$$\begin{aligned} a^2 &= \beta_k^{(n)} + s \\ b &= 2i\beta_k^{(n)}\sqrt{2\rho_k^{(n)}} \end{aligned} \qquad (40)$$

Substituting (39) and (40) into the integral portion of (37), we get

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$$\int_0^\infty \exp\left(-u^2(\beta_k^{(n)}+s)\right) I_0\left(2\beta_k^{(n)}\sqrt{2\rho_k^{(n)}}\ u\right) u\ du$$

$$= \int_0^\infty \exp\left(-u^2a^2\right) I_0(ibu)\ u\ du$$

$$= \frac{1}{2a^2}\ e^{-b^2/4a^2} \tag{41}$$

$$= \frac{1}{2(\beta_k^{(n)}+s)}\ \exp\left(\frac{8(\beta_k^{(n)})^2\rho_k^{(n)}}{4(\beta_k^{(n)}+s)}\right)$$

$$= \frac{1}{2(\beta_k^{(n)}+s)}\ \exp\left(\frac{2(\beta_k^{(n)})^2\rho_k^{(n)}}{\beta_k^{(n)}+s}\right)$$

Putting (41) into (37), we get

$$F_{Z_{1k}^{(n)}}(s) = \frac{\beta_k^{(n)}}{\beta_k^{(n)}+s}\ \exp\left(-2\rho_k^{(n)}\beta_k^{(n)} + \frac{2(\beta_k^{(n)})^2\rho_k^{(n)}}{\beta_k^{(n)}+s}\right)$$

$$= \frac{\beta_k^{(n)}}{\beta_k^{(n)}+s}\ \exp\left(\frac{-2\rho_k^{(n)}\beta_k^{(n)}(\beta_k^{(n)}+s) + 2(\beta_k^{(n)})^2\rho_k^{(n)}}{\beta_k^{(n)}+s}\right) \tag{42}$$

$$= \frac{\beta_k^{(n)}}{\beta_k^{(n)}+s}\ \exp\left(-\frac{2\rho_k^{(n)}\beta_k^{(n)}s}{\beta_k^{(n)}+s}\right)$$

Since all the hops are independent, we obtain the conditional probability density function for the decision variable Z_1 given that $i_1, i_2, \ldots, i_j$ hops of the j bits have interference as

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$$f_{Z_1}(z_1 \mid i_1,i_2,\ldots,i_j) = \left[\left(f_{Z_{1k}^{(1)}}(z_{1k}^{(1)})\right)^{\otimes i_1} \otimes \left(f_{Z_{1k}^{(2)}}(z_{1k}^{(2)})\right)^{\otimes(L-i_1)}\right] \otimes$$

$$\left[\left(f_{Z_{1k}^{(1)}}(z_{1k}^{(1)})\right)^{\otimes i_2} \otimes \left(f_{Z_{1k}^{(2)}}(z_{1k}^{(2)})\right)^{\otimes(L-i_2)}\right] \ldots \otimes \left[\left(f_{Z_{1k}^{(1)}}(z_{1k}^{(1)})\right)^{\otimes i_j} \otimes \left(f_{Z_{1k}^{(2)}}(z_{1k}^{(2)})\right)^{\otimes(L-i_j)}\right] \qquad (43)$$

where $a \otimes b$ implies the convolution of a and b, and $a^{\otimes c}$ represents a c-fold convolution of a. Equation (43) simplifies to

$$f_{Z_1}(z_1 \mid i_1,i_2,\ldots,i_j) = \left(f_{Z_{1k}^{(1)}}(z_{1k}^{(1)})\right)^{\otimes(i_1+i_2+\ldots+i_j)}$$

$$\otimes \left(f_{Z_{1k}^{(2)}}(z_{1k}^{(2)})\right)^{\otimes[(L-i_1)+(L-i_2)+\ldots+(L-i_j)]} \qquad (44)$$

$$= \left(f_{Z_{1k}^{(1)}}(z_{1k}^{(1)})\right)^{\otimes\sum_{p=1}^{j} i_p} \otimes \left(f_{Z_{1k}^{(2)}}(z_{1k}^{(2)})\right)^{\otimes\left[jL-\sum_{p=1}^{j} i_p\right]}$$

In order to determine the convolution, we will multiply in the s domain. That is, from (42) we get the Laplace transform of the conditional probability density function of Z_1 given that i_1, i_2, $\ldots\ldots$, i_j hops have interference

$$F_{Z_1}(s \mid \underline{i}) = \left[\frac{\beta_k^{(1)}}{\beta_k^{(1)}+s} \exp\left(-\frac{2\rho_k^{(1)}\beta_k^{(1)}s}{\beta_k^{(1)}+s}\right)\right]^{\sum_{p=0}^{j} i_p}$$

$$\left[\frac{\beta_k^{(2)}}{\beta_k^{(2)}+s} \exp\left(-\frac{2\rho_k^{(2)}\beta_k^{(2)}s}{\beta_k^{(2)}+s}\right)\right]^{jL-\sum_{p=0}^{j} i_p} \qquad (45)$$

The above Laplace transform cannot be inverted analytically except when either all hops are jammed or no hops are jammed. In these two cases, the inverse Laplace transform of (45) is

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$$\mathcal{L}^{-1}\left(F_{Z_1}(s)\right) = \mathcal{L}^{-1}\left[\left(F_{Z_{1k}}^{(n)}(s)\right)^{jL}\right]$$

$$= (\beta_k^{(n)})^{jL} \exp\left(-2jL\beta_k^{(n)}\rho_k^{(n)}\right)$$

$$\mathcal{L}^{-1}\left[\left(\frac{1}{s+\beta_k^{(n)}}\right)^{jL} \exp\left(\frac{2jL(\beta_k^{(n)})^2\rho_k^{(n)}}{s+\beta_k^{(n)}}\right)\right] \tag{46}$$

$$= (\beta_k^{(n)})^{jL} \exp\left(-2jL\beta_k^{(n)}\rho_k^{(n)}\right) \exp\left(-\beta_k^{(n)}z_1\right)$$

$$\mathcal{L}^{-1}\left[\left(\frac{1}{s}\right)^{jL} \exp\left(\frac{2jL(\beta_k^{(n)})^2\rho_k^{(n)}}{s}\right)\right]$$

From [Ref. 11 : pp 59, eq 81], we get

$$\mathcal{L}^{-1}\left(\frac{1}{s^\mu} e^{k/s}\right) = \left(\frac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}\left(2\sqrt{kt}\right) \tag{47}$$

Let

$$\begin{aligned} k &= 2jL(\beta_k^{(n)})^2 \rho_k^{(n)} \\ \mu &= jL \end{aligned} \tag{48}$$

Substituting this into the inverse Laplace transform on the right hand side of (46), we get

$$\mathcal{L}^{-1}\left[\left(\frac{1}{s}\right)^{jL} \exp\left(\frac{2jL(\beta_k^{(n)})^2\rho_k^{(n)}}{s}\right)\right] \approx \left(\frac{z_1}{2jL(\beta_k^{(n)})^2\rho_k^{(n)}}\right)^{\frac{jL-1}{2}} \tag{49}$$

$$I_{jL-1}\left(2\sqrt{2jL(\beta_k^{(n)})^2\rho_k^{(n)}z_1}\right)$$

Therefore, substituting (49) into (46), we get the probability density function of Z_1 for either no jamming or full band jamming as

$$
\begin{aligned}
f_{Z_1}(z_1) &= \mathcal{L}^{-1}\left[\left(F_{Z_{1k}^{(n)}}(s)\right)^{jL}\right] \\
&= (\beta_k^{(n)})^{jL} \exp\left(-2jL\beta_k^{(n)}\rho_k^{(n)} - \beta_k^{(n)}z_1\right) \\
&\quad \left(\frac{z_1}{2jL(\beta_k^{(n)})^2\rho_k^{(n)}}\right)^{\frac{jL-1}{2}} I_{jL-1}\left(2\sqrt{2jL(\beta_k^{(n)})^2\rho_k^{(n)}z_1}\right)
\end{aligned}
\tag{50}
$$

$$
\begin{aligned}
f_{Z_1}(z_1) &= (\beta_k^{(n)})^{jL} \exp\left[-\beta_k^{(n)}\left(z_1 + 2jL\rho_k^{(n)}\right)\right] (z_1)^{\frac{jL-1}{2}} \\
&\quad \left(\frac{1}{2jL\rho_k^{(n)}}\right)^{\frac{jL-1}{2}} \frac{1}{(\beta_k^{(n)})^{jL-1}} I_{jL-1}\left(2\beta_k^{(n)}\sqrt{2jL\rho_k^{(n)}z_1}\right)
\end{aligned}
\tag{51}
$$

This can be simplified to

$$
\begin{aligned}
f_{Z_1}(z_1) &= \frac{\beta_k^{(n)}(z_1)^{\frac{jL-1}{2}}}{\left(2jL\rho_k^{(n)}\right)^{\frac{jL-1}{2}}} \exp\left[-\beta_k^{(n)}\left(z_1 + 2jL\rho_k^{(n)}\right)\right] \\
&\quad I_{jL-1}\left(2\beta_k^{(n)}\sqrt{2jL\rho_k^{(n)}z_1}\right)
\end{aligned}
\tag{52}
$$

Therefore, the probability density function of Z_1 has been derived.

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3. Probability Density Function of Z_1 with Rayleigh Fading

For the special case of Rayleigh fading, $\rho_k \to 0$, and the Laplace transform of the conditional probability density function of Z_1 given i_1, i_2, ..., i_j given by (46) reduces to

$$F_{Z_1}(s \mid \underline{i}) = \left[\frac{\beta_k^{(1)}}{\beta_k^{(1)}+s} \right]^{\sum\limits_{p=0}^{j} i_p} \left[\frac{\beta_k^{(2)}}{\beta_k^{(2)}+s} \right]^{jL-\sum\limits_{p=0}^{j} i_p} . \tag{53}$$

Depending on the values of i_p, the powers of the bracketed terms can theoretically vary up to infinity. Furthermore, we know that $\beta_k^{(n)}$ is a constant for each hop and each bit. Therefore, we let

$$m = \sum_{p=1}^{j} i_p$$

$$n = jL-\sum_{p=1}^{j} i_p \tag{54}$$

$$a = \beta_k^{(1)}$$

$$b = \beta_k^{(2)}$$

and (53) simplifies to

$$\begin{aligned} F_{Z_1}(s \mid \underline{i}) &= \left(\frac{a}{s+a} \right)^m \left(\frac{b}{s+b} \right)^n \\ &= a^m \, b^n \left(\frac{1}{(s+a)^m} \; \frac{1}{(s+b)^n} \right) \end{aligned} \tag{55}$$

By using partial fractions for the above equation, we get

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$$F_{Z_1}(s \mid i) = a^m b^n \left[\frac{K_{11}}{(s+a)} + \frac{K_{12}}{(s+a)^2} + \frac{K_{13}}{(s+a)^3} + \ldots + \frac{K_{1m}}{(s+a)^m} + \right.$$
$$\left. \frac{K_{21}}{(s+b)} + \frac{K_{22}}{(s+b)^2} + \frac{K_{23}}{(s+b)^3} + \ldots + \frac{K_{2n}}{(s+b)^n} \right] \qquad (56)$$

Using Heaviside's Expansion Theorem for partial fractions [Ref. 12 : pp 174, eq 7-83], we get

$$K_{jx} = \frac{1}{(r-x)!} \frac{d^{r-x}}{ds^{r-x}} R_j(s) \Big|_{s=-a;-b} \qquad (57)$$

where $j = 1, 2$

r = highest power of the denominators (i.e., if $j=1$, $r=m$; and if $j=2$, $r=n$)

x = the xth term of the expansion

and

$$R_1(s) = \frac{1}{(s+b)^n}$$
$$\qquad (58)$$
$$R_2(s) = \frac{1}{(s+a)^m}$$

Therefore,

$$K_{1m}(s) = R_1(s)\big|_{s=-a} = \frac{1}{(b-a)^n}$$
$$\qquad (59)$$
$$K_{2n}(s) = R_2(s)\big|_{s=-b} = \frac{1}{(a-b)^m}$$

Furthermore,

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$$K_{1(m-1)} = \frac{1}{(m-m+1)!} \frac{d}{ds}R_1(s)\Big|_{s=-a}$$

$$= -\frac{n}{(s+b)^{n+1}}\Big|_{s=-a} \tag{60}$$

$$= -\frac{n}{(b-a)^{n+1}}$$

$$K_{1(m-2)} = \frac{1}{2!} \frac{d^2}{ds^2}R_1(s)\Big|_{s=-a}$$

$$= \frac{1}{2!}\left(\frac{n(n+1)}{(b-a)^{n+2}}\right) \tag{61}$$

$$K_{1(m-i)} = \frac{1}{i!} \frac{d^i}{ds^i}R_1(s)\Big|_{s=-a}$$

$$= \frac{1}{i!}\left(\frac{(-1)^i\, n(n+1)(n+2).....(n+i-1)}{(b-a)^{n+i}}\right) \tag{62}$$

$$K_{i1} = \frac{1}{(m-1)!}\left(\frac{(-1)^{m-1}\, n(n+1)(n+2).....(n+m-2)}{(b-a)^{n+m-1}}\right) \tag{63}$$

Similarly,

$$K_{2(n-1)} = \frac{1}{(n-n+1)!} \frac{d}{ds}R_2(s)\Big|_{s=-b}$$

$$= -\frac{m}{(s+a)^{m+1}}\Big|_{s=-b} \tag{64}$$

$$= -\frac{m}{(a-b)^{m+1}}$$

$$K_{2(n-2)} = \frac{1}{2!} \frac{d^2}{ds^2}R_2(s)\Big|_{s=-b}$$

$$= \frac{1}{2!}\left(\frac{m(m+1)}{(a-b)^{m+2}}\right) \tag{65}$$

$$K_{2(n-i)} = \frac{1}{i!} \frac{d^i}{ds^i}R_2(s)\Big|_{s=-b}$$

$$= \frac{1}{i!}\left(\frac{(-1)^i\ m(m+1)(m+2).....(m+i-1)}{(a-b)^{m+i}}\right) \tag{66}$$

$$K_{21} = \frac{1}{(n-1)!}\left(\frac{(-1)^{n-1}\ m(m+1)(m+2).....(n+m-2)}{(a-b)^{n+m-1}}\right) \tag{67}$$

Using the following Laplace transform pairs,

$$f(t) = t^n \quad \leftrightarrow \quad F(s) = \frac{n!}{s^{n+1}}$$

$$f(t) = e^{-at} \quad \leftrightarrow \quad F(s) = \frac{1}{s+a} \tag{68}$$

we get,

$$\mathcal{L}^{-1}\left(\frac{a^m}{(s+a)^m}\right) = a^m\ e^{-at}\ \frac{t^{m-1}}{(m-1)!}$$

$$\mathcal{L}^{-1}\left(\frac{b^n}{(s+b)^n}\right) = b^n\ e^{-bt}\ \frac{t^{n-1}}{(n-1)!} \tag{69}$$

Therefore, the inverse Laplace transform (53) is found to be

$$f_{Z_1}(z_1|\text{ii}) = \left[K_{11} + K_{12}z_1 + \ldots + K_{1m}\frac{z_1^{m-1}}{(m-1)!} \right]$$
$$\exp(-\beta_{kl}^{(1)} z_1) \, u(z_1) \; +$$

$$\left[K_{21} + K_{22}z_1 + \ldots + K_{2n}\frac{z_1^{n-1}}{(n-1)!} \right]$$
$$\exp(-\beta_{kl}^{(2)} z_2) \, u(z_1) \tag{70}$$

Using $m=1$ and $n=2$ in (70), we get

$$K_{11} = \frac{1}{(b-a)^2}$$

$$K_{22} = \frac{1}{(a-b)} \tag{71}$$

$$K_{21} = -\frac{1}{(a-b)^2}$$

Next, using $m=2$ and $n=2$ in (70), we get

$$K_{12} = \frac{1}{(b-a)^2}$$

$$K_{11} = -\frac{2}{(b-a)^3}$$

$$K_{22} = \frac{1}{(a-b)^2} \tag{72}$$

$$K_{21} = -\frac{2}{(a-b)^3}$$

It can be seen from the above that the results are as expected in [Ref. 1 : eq 15].

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C. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLE Z_2

In order to get the probability density function of Z_{2k}, we let the direct (ρ_k) and diffuse (ξ_k) components be zero and $\beta_k^{(n)} = \frac{1}{2}$ in (42). This result is then raised to the $(jL)^{\text{th}}$ power to yield

$$
\begin{aligned}
F_{Z_2}(s) &= \left(\frac{\beta_k^{(n)}}{\beta_k^{(n)} + s} \right)^{jL} \\
&= \left(\frac{1}{2(s + \frac{1}{2})} \right)^{jL}
\end{aligned}
\tag{73}
$$

which is inverted easily to obtain

$$
f_{Z_2}(z_2) = \frac{(z_2/2)^{jL-1}}{2(jL-1)!} \exp(-\frac{z_2}{2})
\tag{74}
$$

The dramatic simplification in obtaining the probability density function for Z_2 occurs due to the normalization which makes the output of the branch that does not contain the signal independent of whether the hop is jammed or not. Next, we need to find

$$
\int_0^{z_1} f_{Z_2}(z_2) dz_2 = \int_0^{z_1} \frac{(z_2/2)^{jL-1}}{2(jL-1)!} \exp\left(-\frac{z_2}{2}\right) dz_2
\tag{75}
$$

If we let $x = z_2/2$, this gives

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$$dx = \frac{dz_2}{2}$$

$$z_2 = 0 \quad \rightarrow \quad x = 0 \tag{76}$$

$$z_2 = z_1 \quad \Rightarrow \quad x = \frac{z_1}{2}$$

Substituting the above into (75), we obtain the simplified integral

$$\int_0^{z_1} f_{z_2}(z_2) \, dz_2 = \int_0^{z_1} \frac{(z_2/2)^{jL-1}}{2(jL-1)!} \, e^{-\frac{z_2}{2}} \, dz_2$$

$$= \int_0^{z_1/2} \frac{x^{jL-1}}{2(jL-1)!} \, e^{-x} \, 2 \, dx \tag{77}$$

$$= \frac{1}{(jL-1)!} \int_0^{z_1/2} x^{jL-1} e^{-x} \, dx$$

From [Ref. 9 : pp 89, eq 2.321.2], we get

$$\int x^n e^{ax} dx = e^{ax} \left(\frac{x^n}{a} + \sum_{k=1}^{n} \frac{n(n-1)\dots(n-k+1)}{a^{k+1}} x^{n-k} \right) \tag{78}$$

If we let $n = (jL-1)$ and $a = -1$ in (78) and use this result in (77), we get

$$\int_0^{z_1} f_{z_2}(z_2)\ dz_2 = \frac{1}{(jL-1)!} \int_0^{z_1/2} x^{jL-1}\ e^{-x}\ dx$$

$$= \left[\frac{e^{-x}}{(jL-1)!}\ \left(-x^{jL-1}\right.\right.$$

$$\left.\left. + \sum_{k=1}^{jL-1} (-1)^k \frac{(jL-1).....(jL-k)}{(-1)^k}\ x^{jL-k-1}\right)\right]_0^{Z_1/2} \tag{79}$$

$$= \left[\frac{e^{-x}}{(jL-1)!}\ \left(-x^{jL-1} - \sum_{k=1}^{jL-1} (jL-1).....(jL-k)\ x^{jL-k-1}\right)\right]_0^{Z_1/2}$$

If we substitute the limits of the integral, we get

$$\int_0^{z_1} f_{z_2}(z_2)\ dz_2 = \frac{e^{-z_1/2}}{(jL-1)!}\ \left[-\left(\frac{z_1}{2}\right)^{jL-1}\right.$$

$$\left. - \sum_{k=1}^{jL-1} (jL-1).....(jL-k)\ \left(\frac{z_1}{2}\right)^{jL-1-k}\right] + \frac{(jL-1)!}{(jL-1)!} \tag{80}$$

This can be simplified to obtain

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$$\int_0^{z_1} f_{Z_2}(z_2)\, dz_2 = 1 - \left[\frac{e^{z_1/2}}{(jL-1)!}\left(\frac{z_1}{2}\right)^{jL-1} \right.$$

$$\left. + e^{-z_1/2} \sum_{k=1}^{jL-1} \frac{(jL-1)\ldots\ldots(jL-k)}{(jL-1)!} \left(\frac{z_1}{2}\right)^{jL-1-k} \right]$$

(81)

$$= 1 - \left[\frac{e^{-z_1/2}}{(jL-1)!} \left(\frac{z_1}{2}\right)^{jL-1} \right.$$

$$\left. + e^{-z1/2} \sum_{k=1}^{jL-1} \frac{(z_1/2)^{jL-1-k}}{(jL-1-k)!} \right]$$

Therefore, the integral of the probability density function for Z_2 is

$$\int_0^{z_1} f_{Z_2}(z_2)\, dz_2 = 1 - \left[e^{-z_1/2} \sum_{k=0}^{jL-1} \frac{(z_1/2)^{jL-1-k}}{(jL-1-k)!} \right]$$

(82)

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III. NUMERICAL RESULTS

A. NUMERICAL PROCEDURE

The computation of the bit error probability requires numerical computation of (7) with the probability density function of Z_1 being computed by finding the inverse Laplace transform of (45) numerically except for the two special cases of either all hops jammed or no hops jammed. The remainder of the integrand of (7) is given by (82). The results of this numerical computation are then substituted into (5).

This result for P_j is then substituted into (3) where the weights (ω_j), the coding free distance (d_{free}), and the number of input bits (k) are used to determine the probability of bit error.

System performance is evaluated for various values of jamming fraction γ, diversity, fading conditions, and values of the bit energy-to-jamming noise density ratio (E_b/N_j). For the Rician case, the ratio of direct-to-diffuse signal energy $(\alpha^2/2\sigma^2)$ is assumed to be the same for each hop k of the bit.

In this study, $E_b/N_0 = 20dB$ is used when $\alpha^2/2\sigma^2 \leq 1$. Otherwise, $E_b/N_0 = 13.35dB$ is used. It is expected that when E_b/N_0 is reduced, performance will improve at lower values of E_b/N_j since E_b/N_0 is more dominant. However, the improvement is inconsequential since for low E_b/N_j system performance with convolutional coding is too poor to be useful. It seems reasonable to assume that the trends obtained with these values of E_b/N_0 will not be significantly affected as E_b/N_0 increases.

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B. PERFORMANCE WITH RAYLEIGH FADING

Figures 2 to 10 shows the performance of the noise normalized receiver with Rayleigh fading. It is observed from these figures that coding improves the performance when E_b/N_I is greater than 10dB.

Figures 2 and 3 shows the performance of the receiver when $\gamma = 1.0$ (full band jamming), 0.25, 0.1 and 0.01 and the number of hops per b. ` $= 1$ with constraint length $\nu = 2$ and 4, respectively. The figures show that the asymptotic probability of bit errors vary from about 10^{-6} ($\nu = 2$) to about 10^{-8} ($\nu = 4$). These asymptotic probability of bit errors are much better than that obtained for uncoded performance (10^{-2}). Comparing Figs. 2 and 3 with Figs. 4 and 5, where L is increased to 2, performance is seen to have improved with the asymptotic bit error probability being 10^{-9} ($\nu = 2$) and 10^{-12} ($\nu = 4$) as compared to 10^{-6} and 10^{-8} when $L = 1$. It is also seen that for Rayleigh fading the worst case jamming strategy to be used is full band jamming.

By increasing ν, performance is further improved. This improvement is much more dramatic at high E_b/N_I as can be seen in Fig. 6. Comparing Fig. 6 with Fig. 7, it is observed that the asymptotic bit error probability is improved either by increasing ν or L. The asymptotic bit error probabilities for various values of ν and L are given in Table 1.

TABLE 1

Constraint Length, ν	Asymptotic Probability of Bit Error	
	L = 1	L = 2
$\nu = 2$	10^{-6}	10^{-9}
$\nu = 4$	10^{-8}	10^{-12}
$\nu = 6$	10^{-11}	$< 10^{-13}$

It can also be seen in Fig. 8 that when E_b/N_i is small ($<$ 15dB) and L is increased that the non-coherent combining losses are dominant and degrade the performance. However, for higher E_b/N_i, the diversity gain is dominant and improves the performance of the receiver.

In Figs. 9 and 10, soft-decision Viterbi decoding is compared to hard-decision Viterbi decoding (for $\nu = 6$). It is observed that soft decision decoding improves performance over hard decision decoding by about 7 to 10dB when E_b/N_i is around 10 to 20dB (Fig. 9). The asymptotic bit error probability for rate = 1/2 is seen to be 10^{-11} as compared to 10^{-4}. Furthermore, for rate = 1/3, the asymptotic bit error probability is 10^{-14} as compared to 10^{-6}. As expected, there is further improvement when L is increased (Fig. 10).

C. PERFORMANCE WITH RICIAN FADING

Figures 11 to 17, Figs. 18 to 24, and Figs. 25 to 26 show the performance of the receiver for a strong direct signal ($\alpha^2/2\sigma^2 = 10$), a weak direct signal ($\alpha^2/2\sigma^2 = 1$), and a very strong direct signal ($\alpha^2/2\sigma^2 = 100$), respectively. It is observed that coding improves performance when E_b/N_I is greater than about 10dB.

Figures 11 and 12 show the performance of the receiver when $\gamma = 1.0$ (full band jamming), 0.25, 0.1 and 0.01 and $\nu = 2$ for $L = 1$ and 2, respectively. The asymptotic bit error probability is about 10^{-6} as compared to 10^{-2} for the uncoded case. The comparable situation, but with a weak direct signal component ($\alpha^2/2\sigma^2 = 1$), is shown in Figs. 18 and 19. In this case, performance is slightly worse as compared to the case with $\alpha^2/2\sigma^2 = 10$. It is observed from all the above cases that the worst case jamming strategy is quite close to full band jamming, even when $L = 1$, except when $\alpha^2/2\sigma^2 = 100$ (Figs. 25 and 26). Even then, partial-band jamming does not significantly degrade performance. Therefore, it is observed that convolutional coding improves the performance of frequency-hopping systems by forcing the jammer to spread its resources over a much wider band as compared to the non-coded system. This conclusion remains valid even when $L=1$. It should also be noted that with a very strong direct signal, the effect of the jammer is much less significant than when the direct signal is weaker for the same E_b/N_I.

By increasing ν (Figs. 13, 14, 20, and 21), it is observed that the performance is improved by about 1dB as ν is increased from 2 to 4 and from 4 to 6. This improvement

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reduces the asymptotic bit error probability for strong direct signal from about 10^{-5} to about 10^{-8} and for the weak direct signal from about 10^{-4} to 10^{-7} when $\nu = 6$.

Figure 15 shows the performance of the receiver when there is an increase in diversity. It is observed that for the strong direct signal ($\alpha^2/2\sigma^2 = 10$) that the non-coherent combining loss is greater than the diversity gain since performance is degraded with an increase in diversity. For the weak direct signal case ($\alpha^2/2\sigma^2 = 1$) (Fig. 22), diversity gain is greater than the non-coherent combining losses when E_b/N_I is greater than 15dB.

With $\nu = 6$ (Figs. 16, 17, 23 and 24), when the rate is reduced from 1/2 to 1/3, it is observed that there is minimal increase in performance when there is a strong direct signal. When there is a weak direct signal, the improvement in performance is about 0.5 to 1.0dB. Comparing these to hard decision decoding, there is an improvement of about 5dB.

From the above discussions, it is concluded that with the implementation of convolutional coding with soft decision Viterbi decoding that performance is generally improved when E_b/N_I is higher than about 10dB. Another conclusion is that convolutional coding and soft decision Viterbi decoding render fast frequency-hopping with $L > 1$ largely unnecessary.

IV. CONCLUSION

When convolutional coding with soft decision Viterbi decoding is employed in conjunction with FFH/BFSK utilizing noise normalization combining, system performance is superior to the performance of the equivalent uncoded system when E_b/N_I is greater than 10dB. Below 10dB the uncoded system performance is too poor for coding to be effective. This improvement in performance is particularly true for Rayleigh fading channels. In addition, at high E_b/N_I, the asymptotic probability of bit error improves dramatically as compared to the uncoded system with the probability of bit error varying from 10^{-6} to 10^{-12} depending on ν, L, and $(\alpha^2/2\sigma^2)$ as compared to about 10^{-3} with no coding. The jamming strategy required to cause worst case receiver performance is full band jamming, even when $L=1$, except for very strong direct signal $(\alpha^2/2\sigma^2 = 100)$. Even then, partial-band jamming does not significantly degrade performance. This is a significant departure from what is observed for the uncoded system. This means that the enemy jammer must spread its jamming power over a much wider bandwidth. It also means that fast frequency-hopping with $L>1$ is largely unnecessary when convolutional coding and soft decision Viterbi decoding are used.

Due to non-coherent combining losses, there is some degradation in performance when the hop per bit ratio is increased and the received signal is at moderate E_b/N_I.

It is found, as expected, that when a stronger code (higher constraint length) is used that performance is improved, especially for high E_b/N_I. Finally, soft decision decoding

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improves the performance over the hard decision decoding by about 4 to 8 dB (depending on the code rate) at moderate E_b/N_1 and has a much lower asymptotic error for high E_b/N_1.

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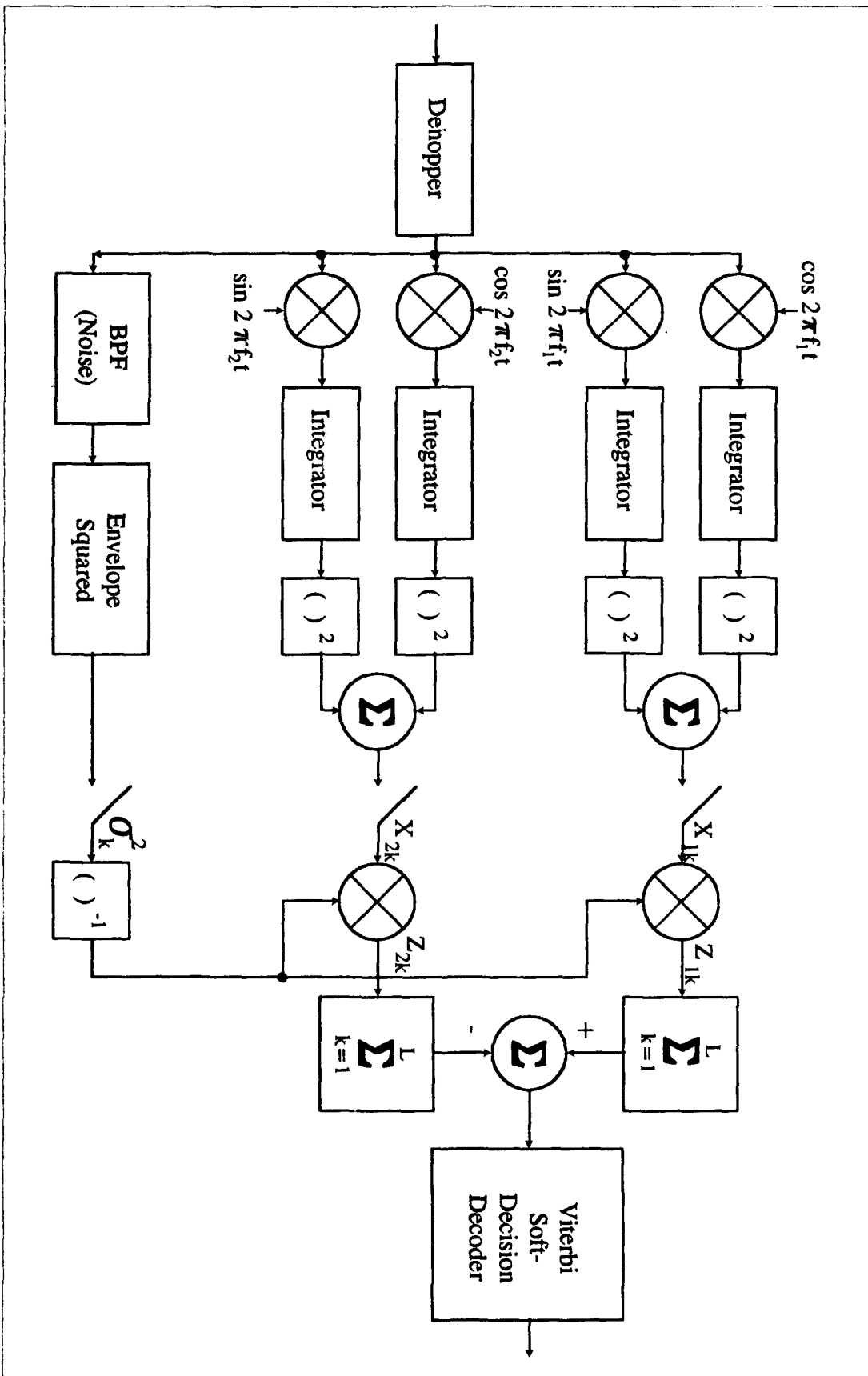
Fig. 1: FFH/BFSK Receiver with Noise Normalization and Soft-Decision Viterbi Decoding

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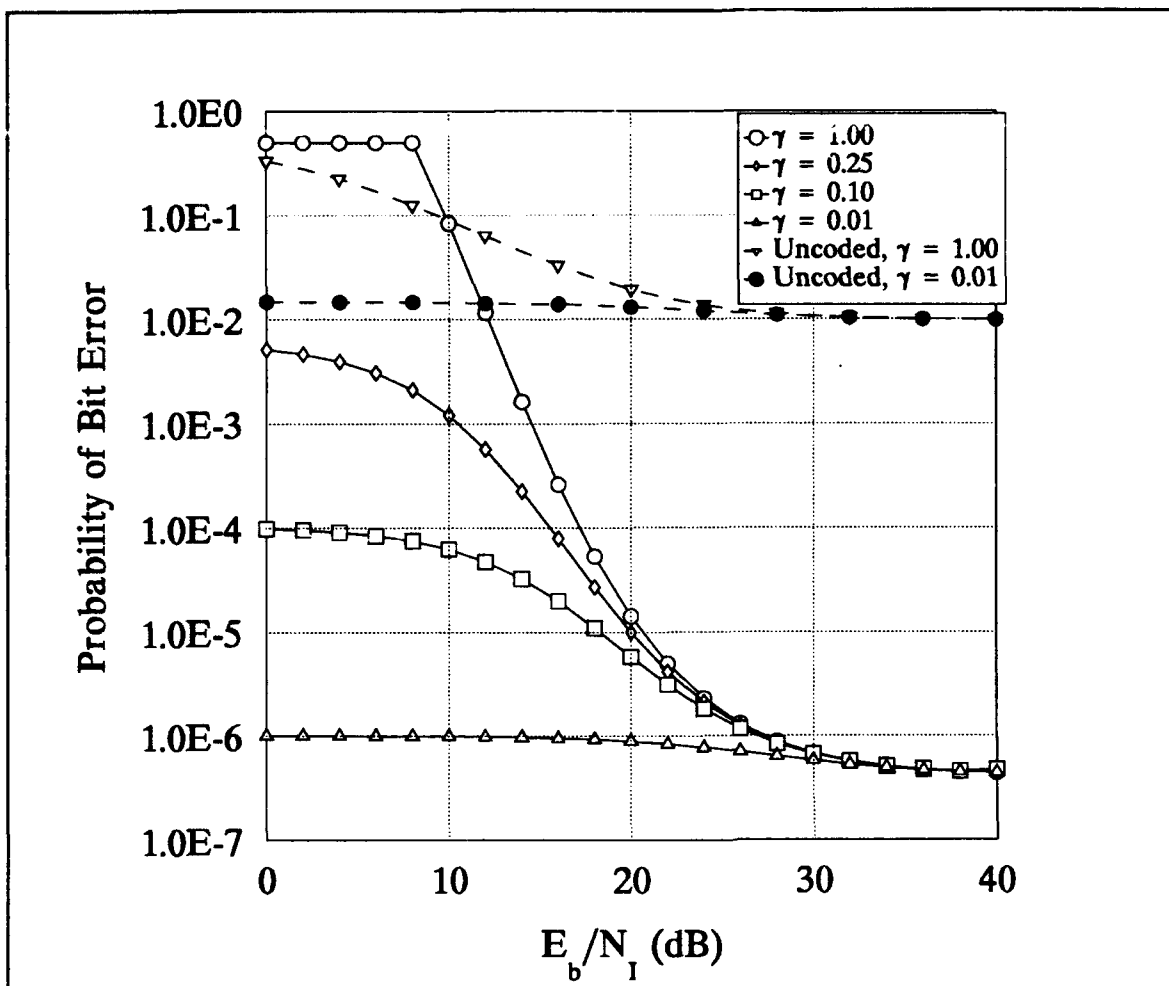
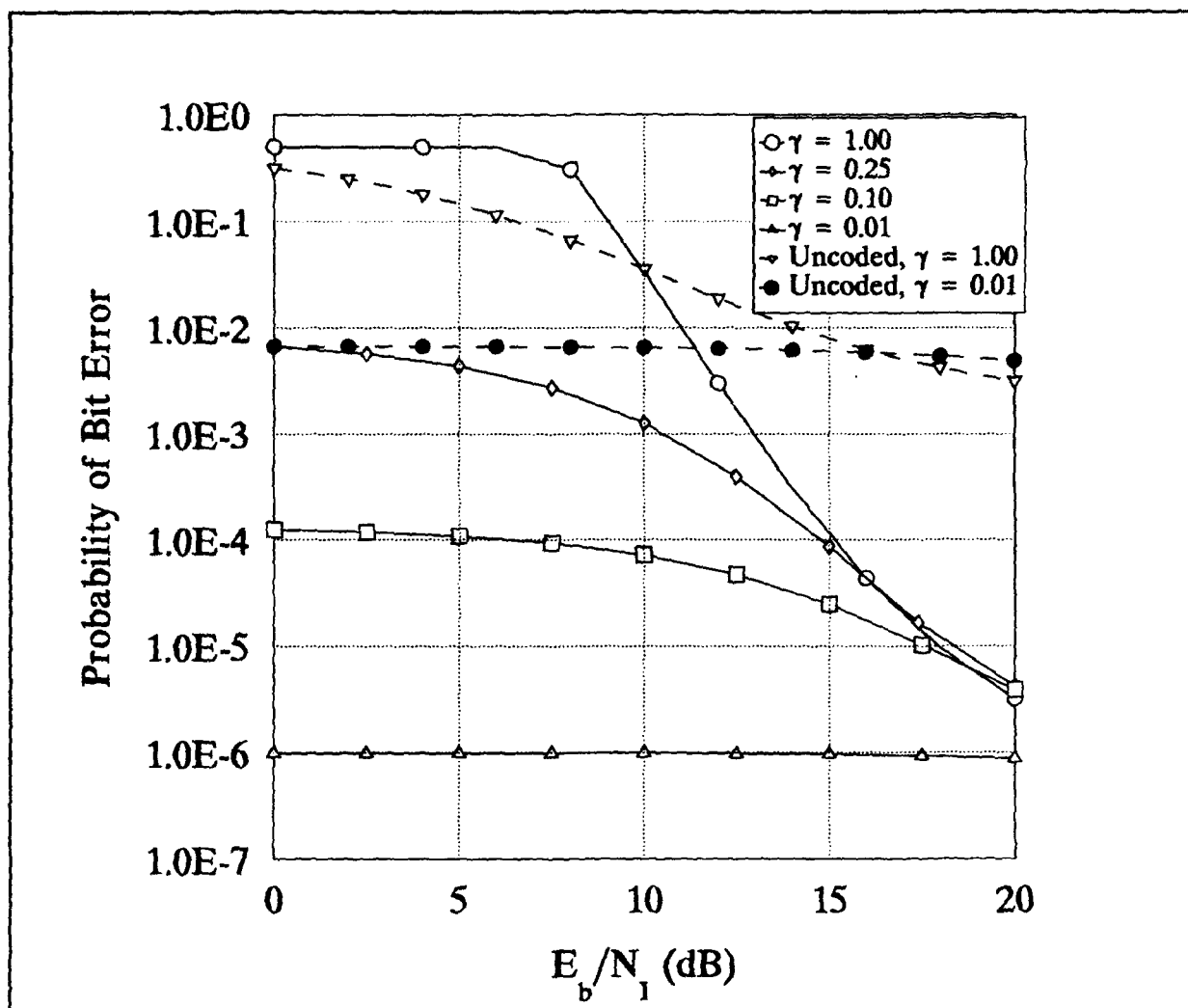**Fig. 2 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for γ = 1.0, 0.25, 0.1 and 0.01 with Rayleigh fading and E_b/N_O = 20.0dB, L = 1, and v = 2.**
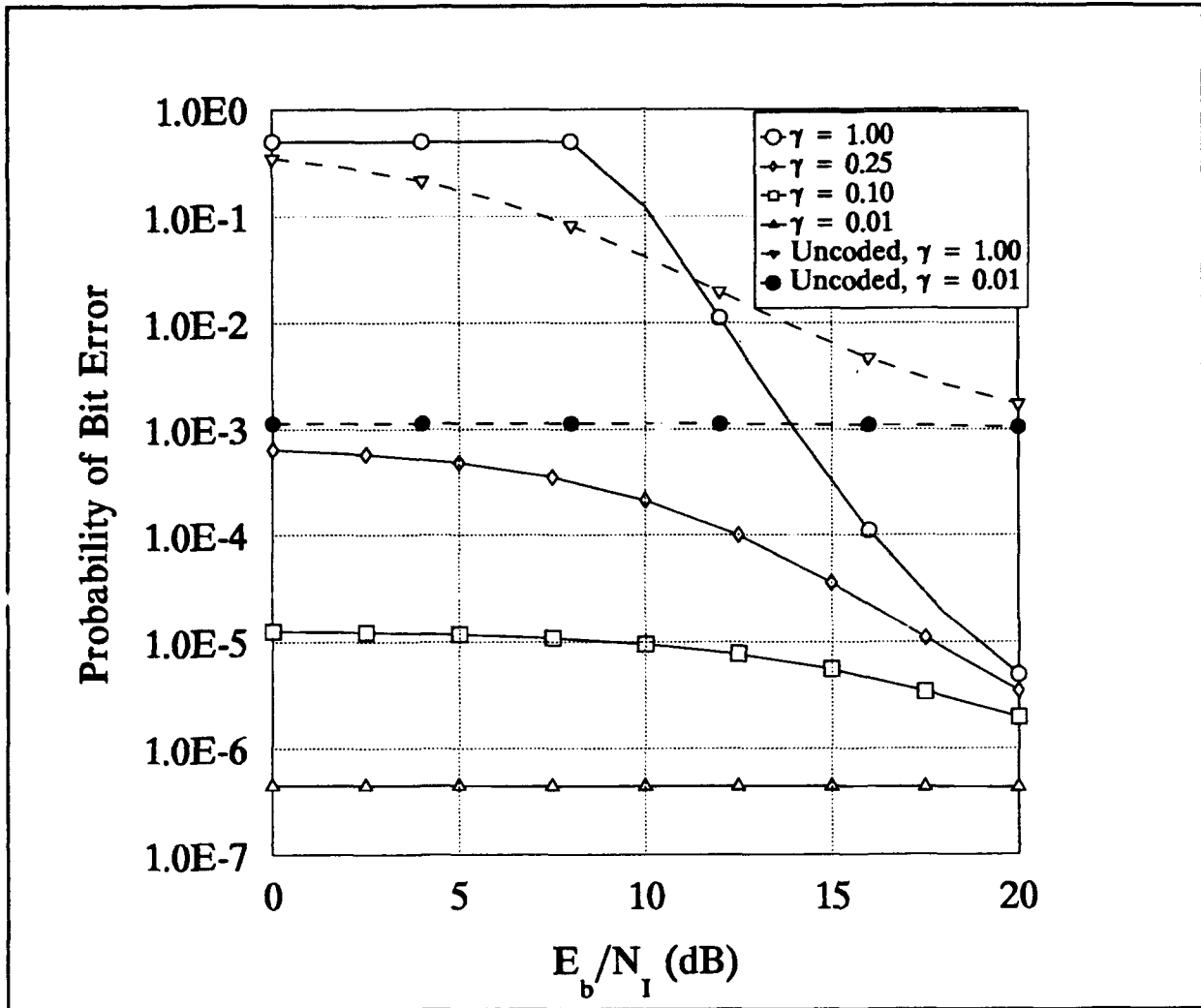
Fig. 3 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for γ = 1.0, 0.25, 0.1 and 0.01 with Rayleigh fading, and E_b/N_O = 20.0dB, L = 1 and ν = 4.

Fig. 4 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for γ = 1.0, 0.25, 0.1 and 0.01 with Rayleigh fading, and E_b/N_O = 20.0dB, L = 2 and v = 2.

Fig. 5 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for γ = 1.0, 0.25, 0.1 and 0.01 with Rayleigh fading, and E_b/N_0 = 20.0dB, L = 2 and v = 4.
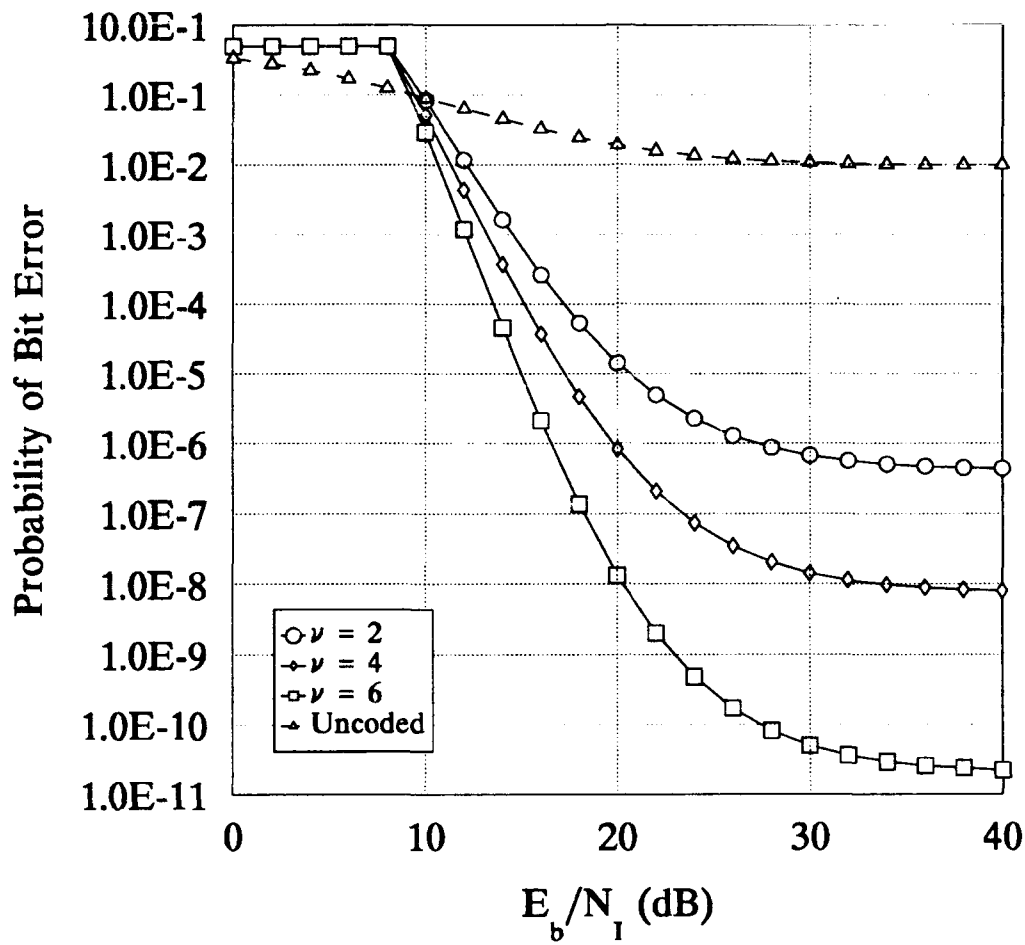
Fig. 6 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for ν = 2, 4 and 6 with Rayleigh fading, full band jamming, E_b/N_0 = 20.0dB, and L = 1.

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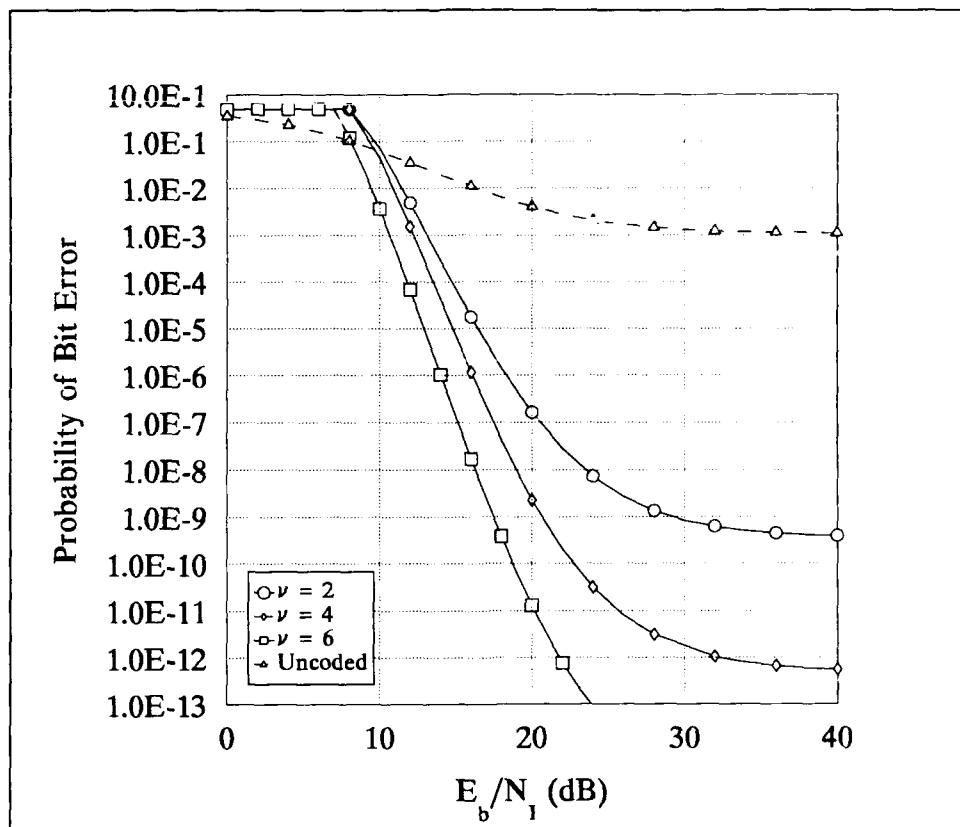
Fig. 7 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decodir
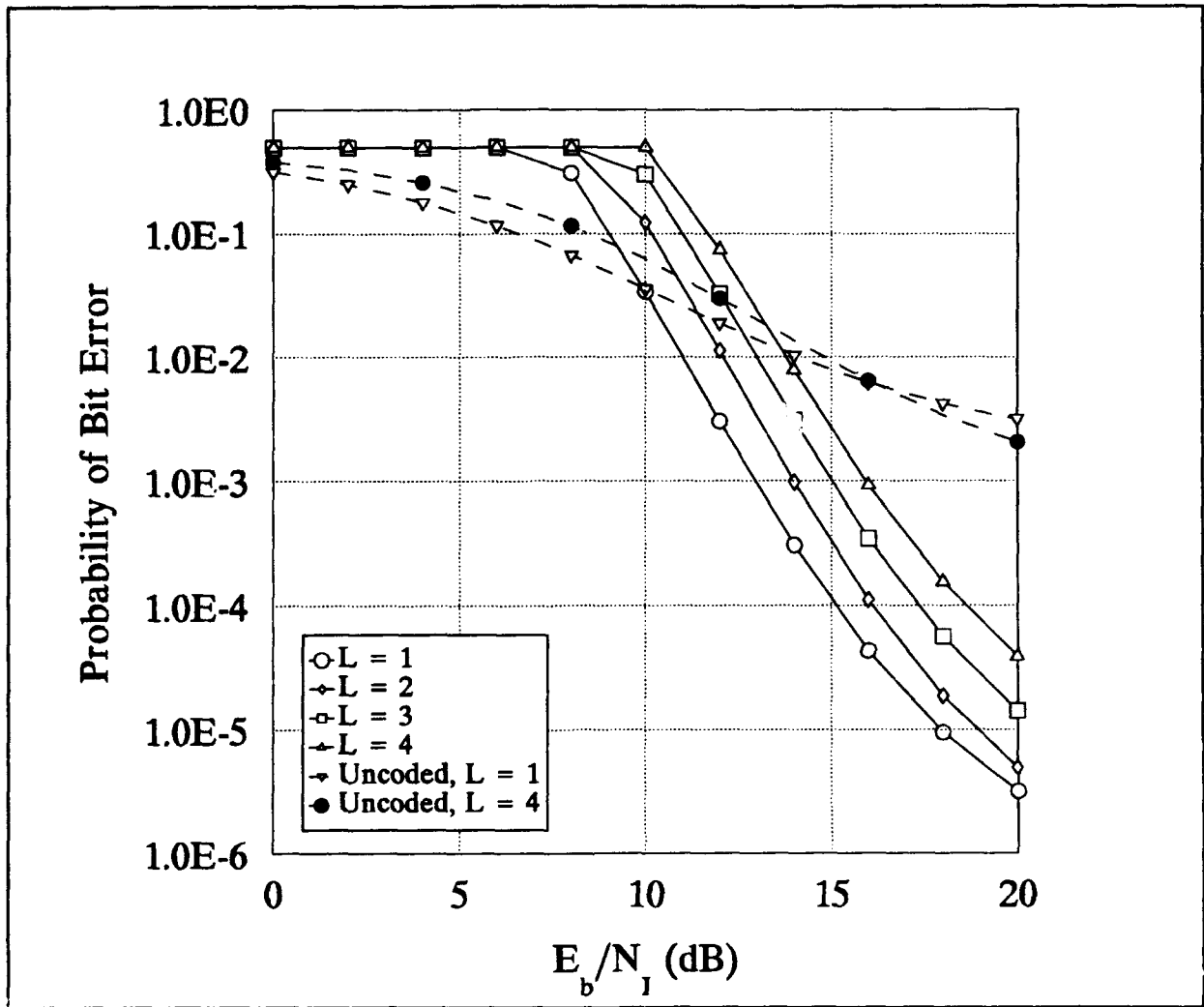for ν = 2, 4 and 6 with Rayleigh fading, full band jamming, E_b/N_0 = 20.0dB, and L = 2

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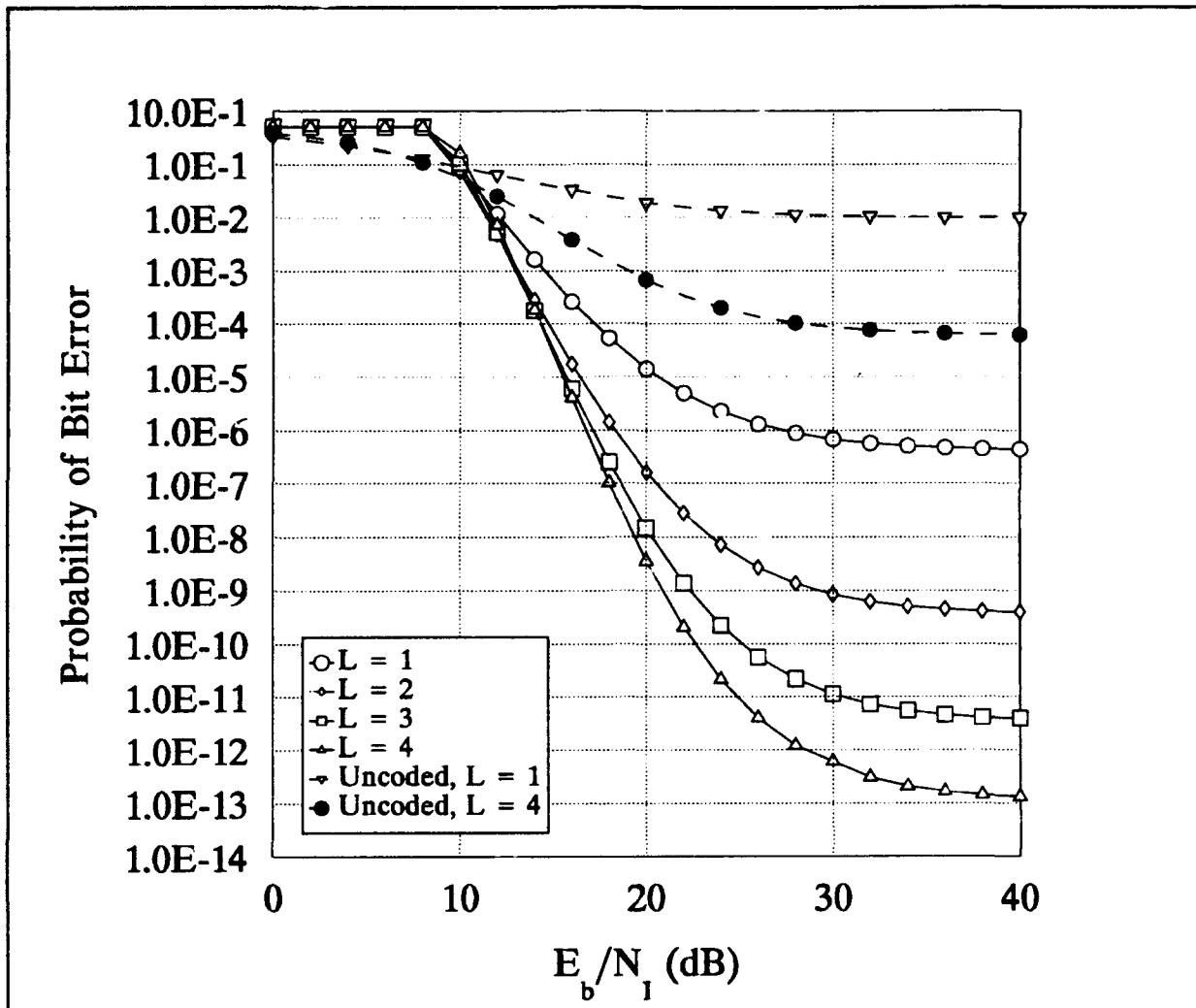
Fig. 8 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for number of hops/bit of L = 1, 2, 3 and 4 with Rayleigh fading, full band jamming, E_b/N_0 = 20.0dB, and ν = 2.

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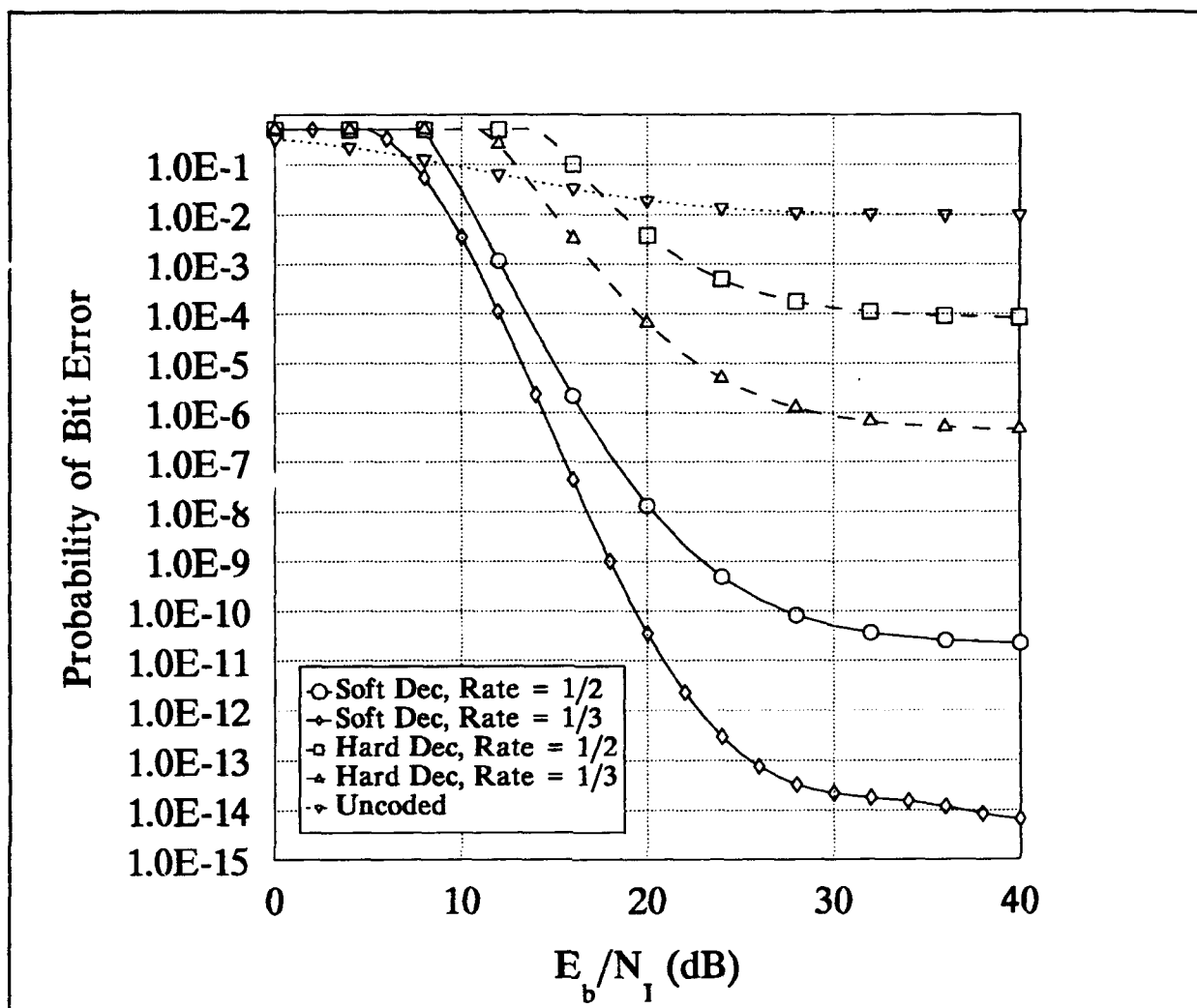
Fig. 9 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate, r = 1/2 and 1/3 with Rayleigh fading, full band jamming, E_b/N_O = 20.0dB, L = 1, and v = 6.
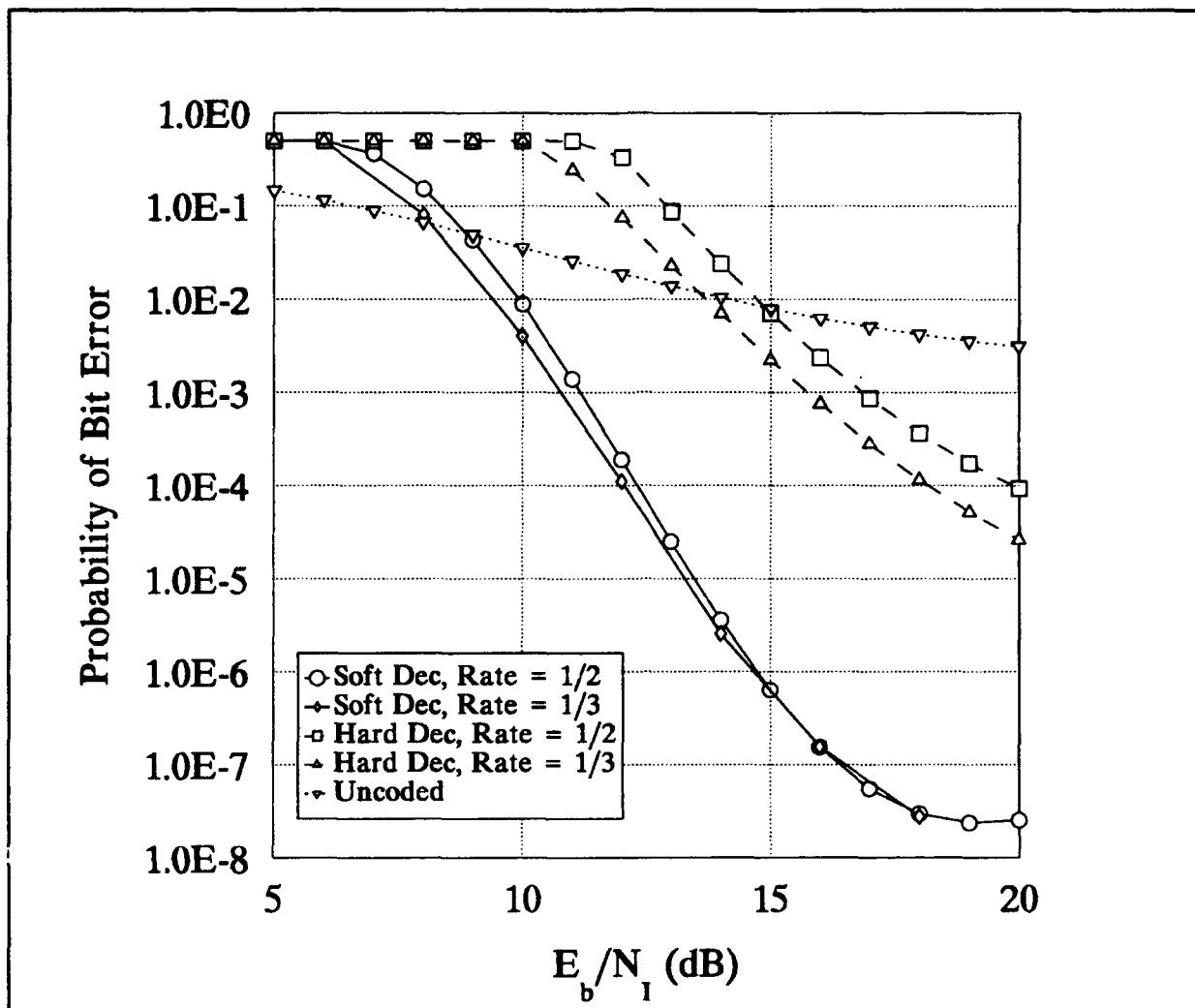
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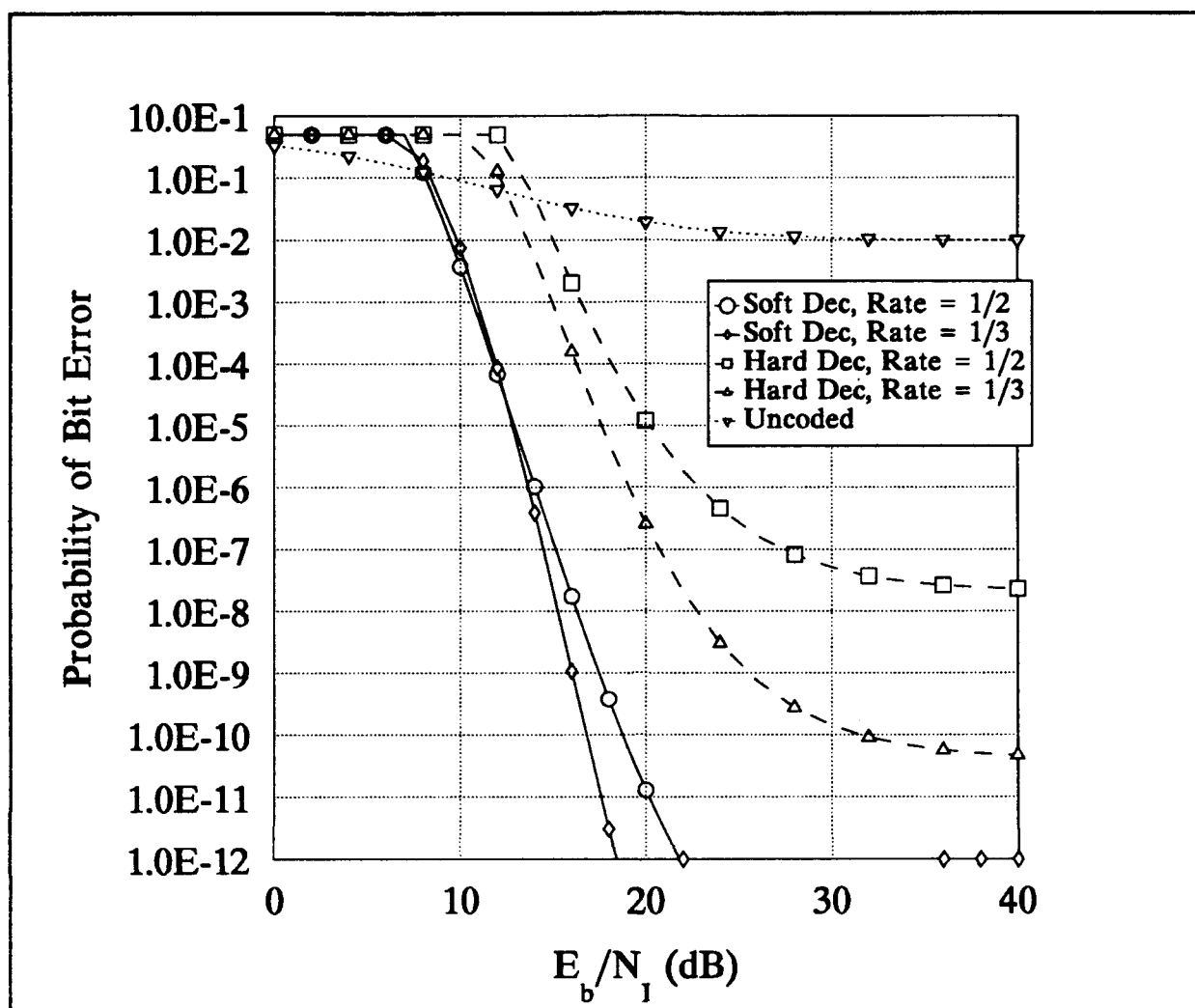
Fig. 10 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate, r = 1/2 and 1/3 with Rayleigh fading, full band jamming, E_b/N_0 = 20.0dB, L = 2, and ν = 6.

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Fig. 11 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for $\gamma = 1.0, 0.25, 0.1$ and 0.01 with Rician fading, a strong direct signal ($\alpha^2/2\sigma^2 = 10$), E_b/N_0 = 13.35dB, L = 1, and $\nu = 2$.

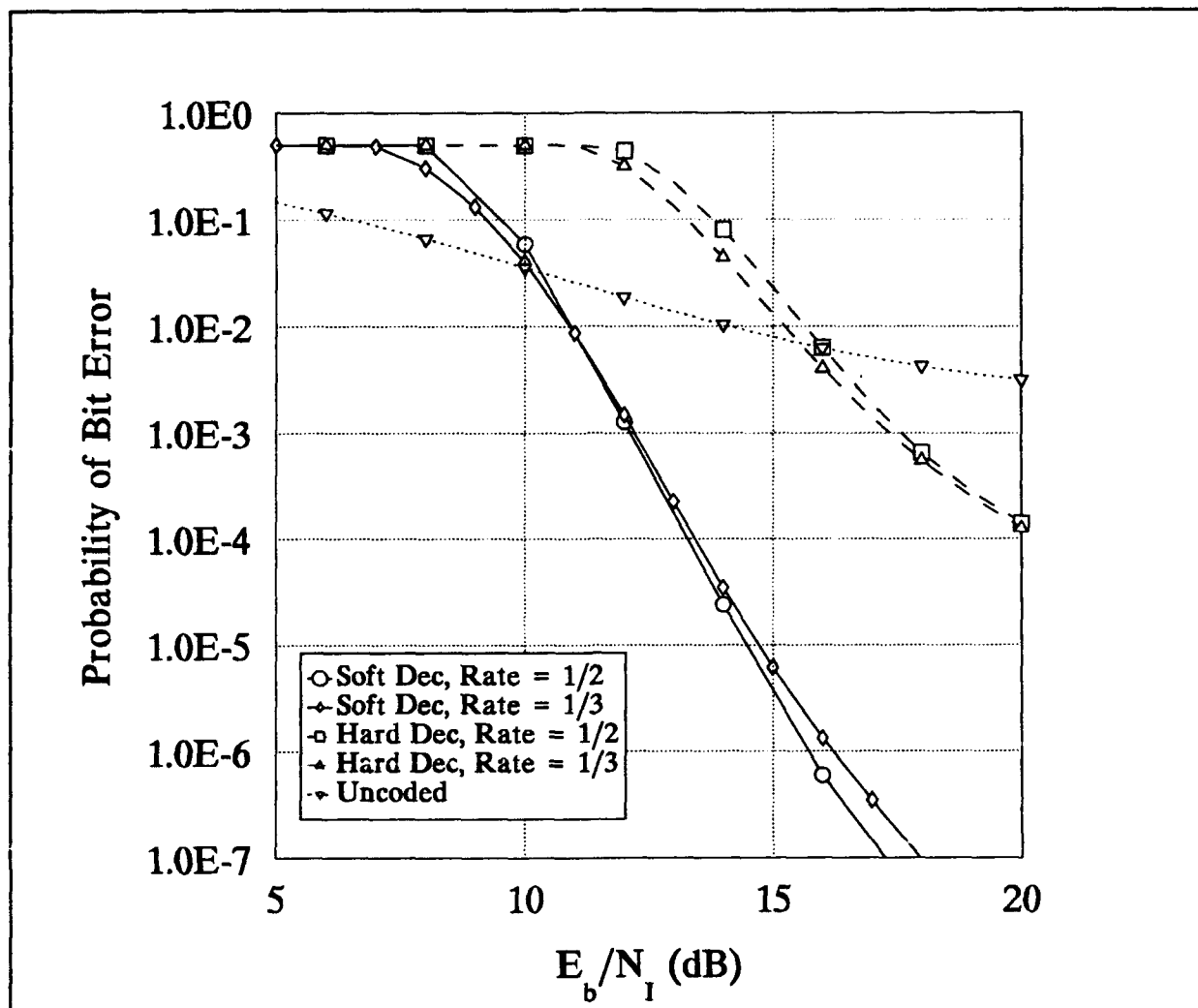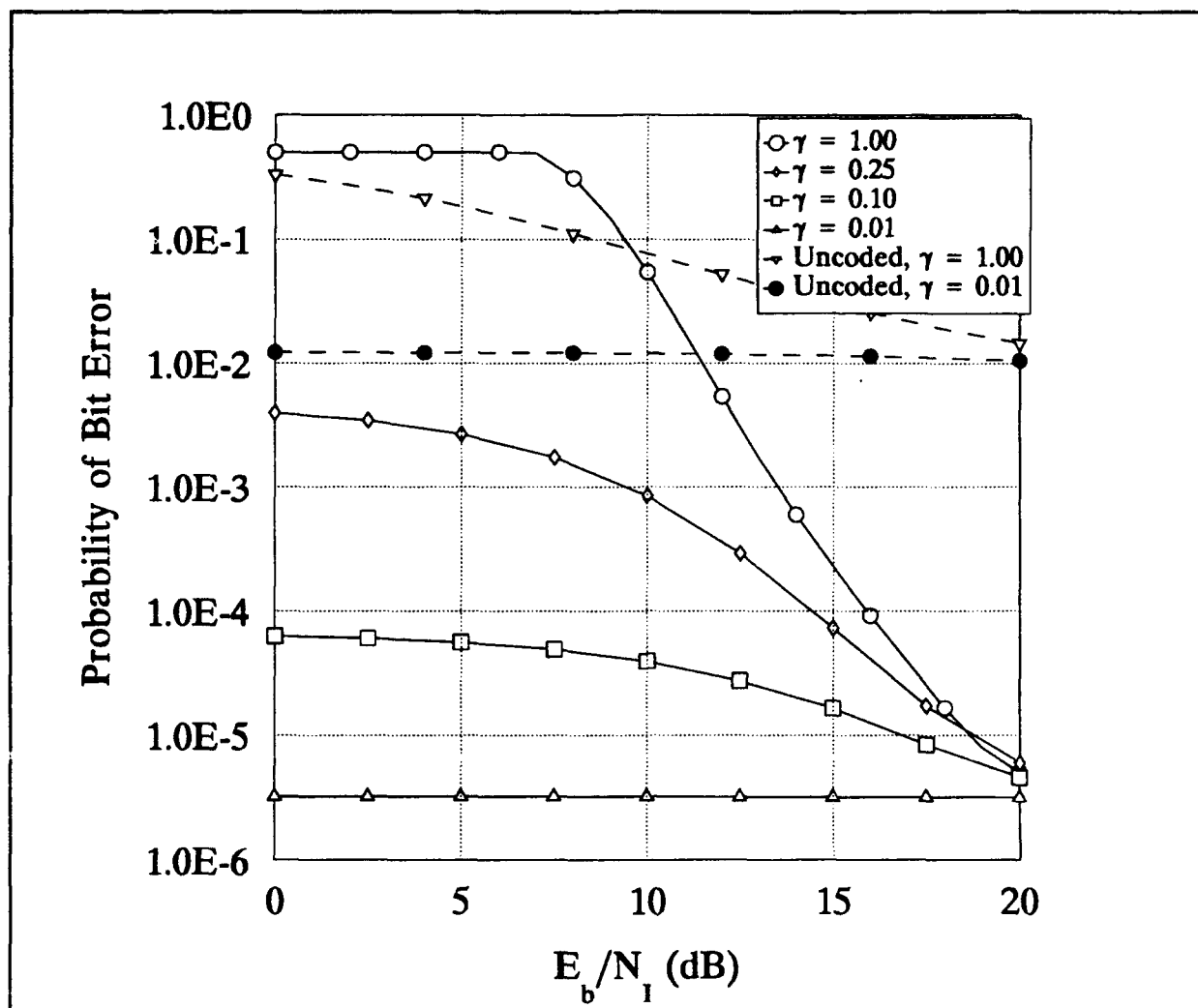**Fig. 12 :** performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for $\gamma = 1.0$, 0.25, 0.1 and 0.01 with Rician fading, a strong direct signal ($\alpha^2/2\sigma^2 = 10$), E_b/N_o = 13.35dB, L = 2, and $\nu = 2$.

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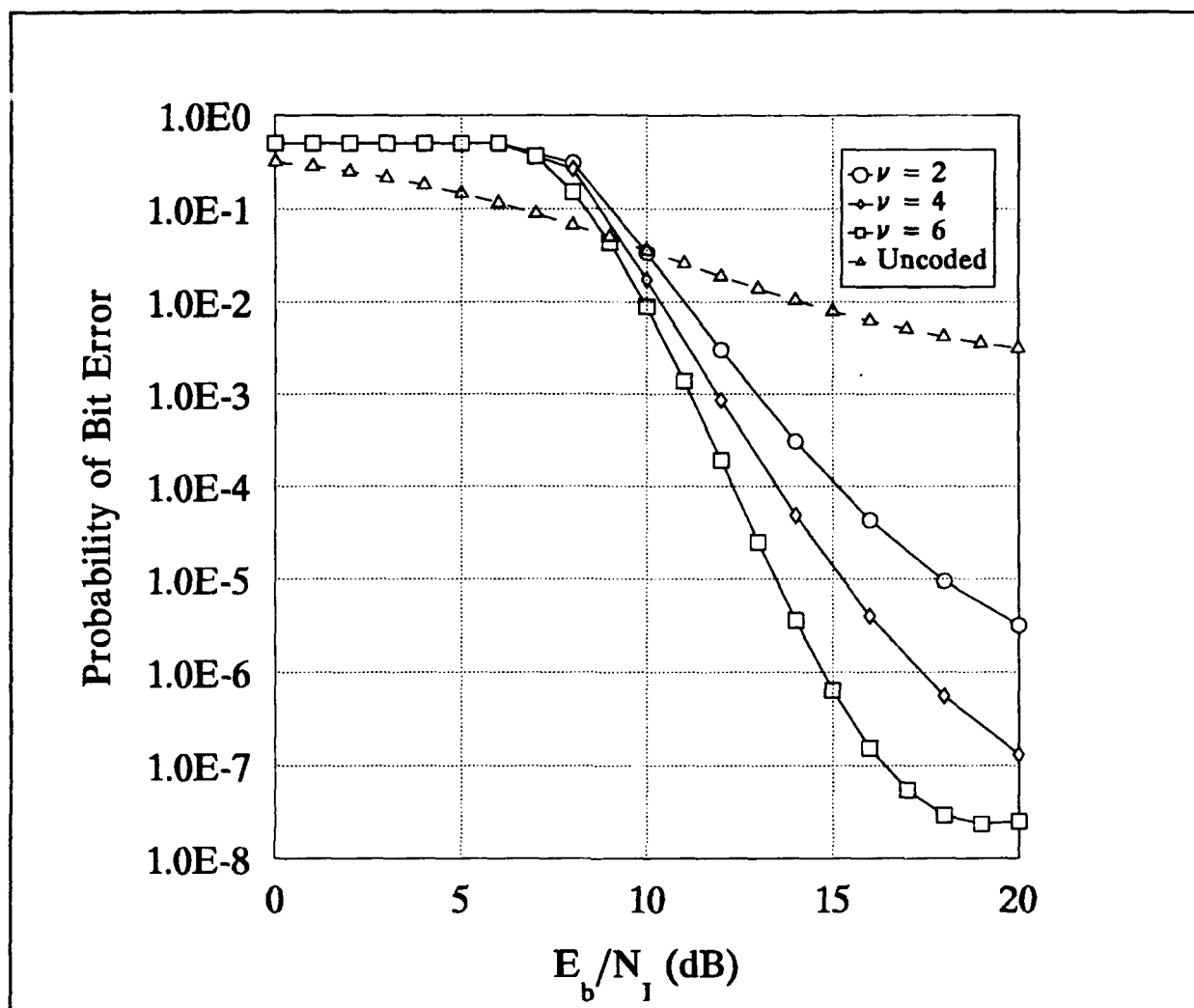
Fig. 13 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for ν = 2, 4 and 6 under Rician fading, a strong direct signal ($\alpha^2/2\sigma^2$ = 10), full band jamming, E_b/N_O = 13.35dB, and L = 1.

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Fig. 14 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for v = 2, 4 and 6 under Rician fading, a strong direct signal ($\alpha^2/2\sigma^2$ = 10), full band jamming, E_b/N_0 = 13.35dB, and L = 2.

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Fig. 15 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for number of hops/bit of L = 1, 2, 3 and 4 with Rician fading, a strong direct signal ($\alpha^2/2\sigma^2$ = 10), full band jamming, E_b/N_O = 13.35dB, and v = 2.

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Fig. 16 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate, r = 1/2 and 1/3 with Rician fading, a strong direct signal ($\alpha^2/2\sigma^2 = 10$), full band jamming, $E_b/N_O = 13.35$dB, L = 1, and $v = 6$.

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Fig. 17 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate, r = 1/2 and 1/3 with Rician fading, a strong direct signal ($\alpha^2/2\sigma^2 = 10$), full band jamming, $E_b/N_O = 13.35$dB, L = 2, and $v = 6$.
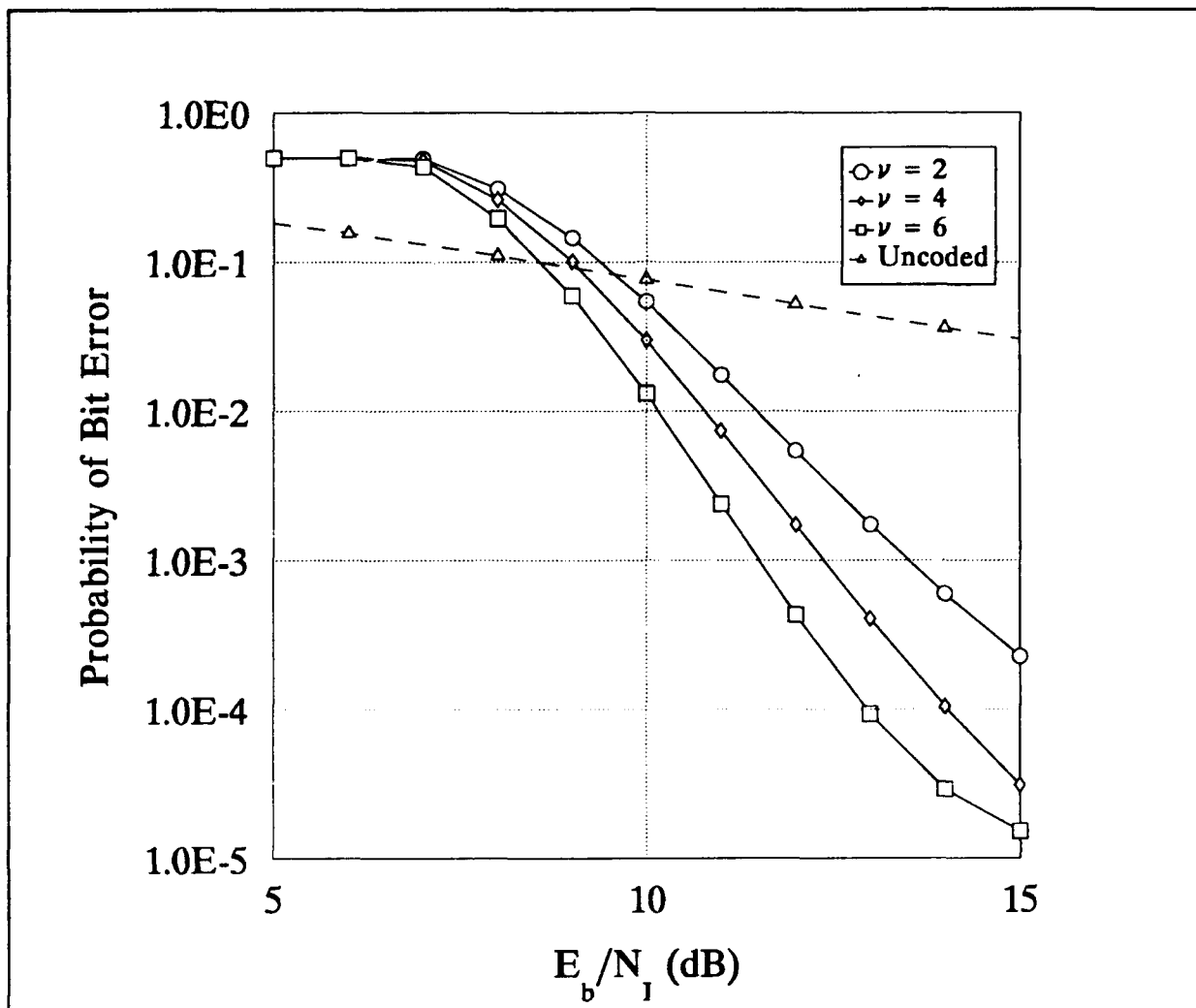
Fig. 18 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for γ = 1.0, 0.25, 0.1 and 0.01 with Rician fading, a weak direct signal ($\alpha^2/2\sigma^2$ = 1), E_b/N_0 = 20.0dB, L = 1, and ν = 2.
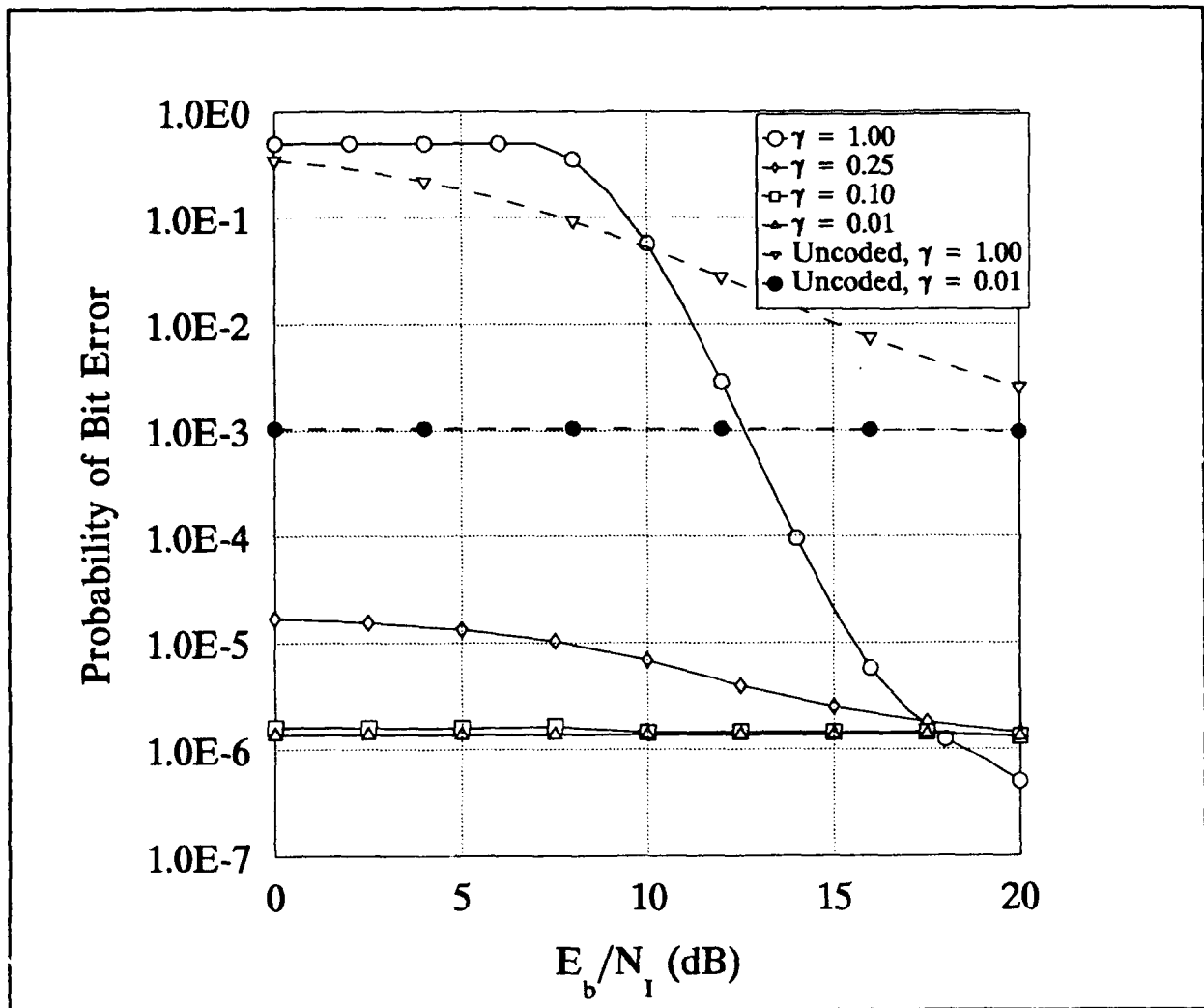
Fig. 19 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for γ = 1.0, 0.25, 0.1 and 0.01 with Rician fading, a weak direct signal ($\alpha^2/2\sigma^2$ = 1), E_b/N_0 = 20.0dB, L = 2, and ν = 2.
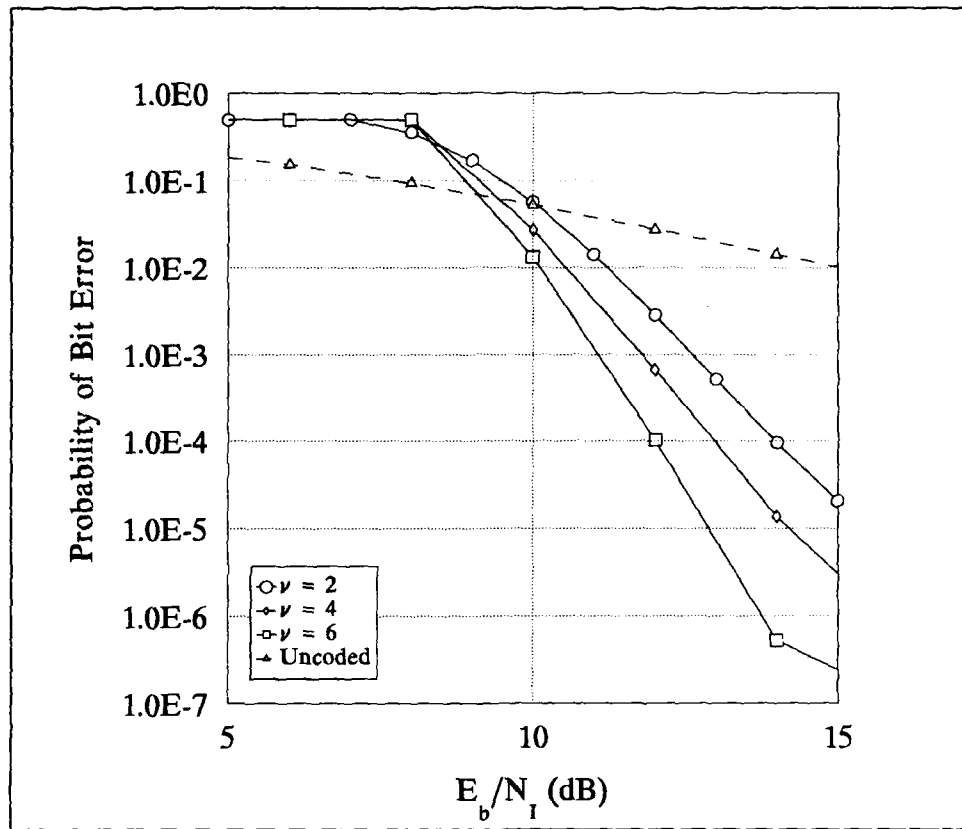
Fig. 20 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for v = 2, 4 and 6 with Rician fading, a weak direct signal ($\alpha^2/2\sigma^2$ = 1), full band jamming, E_b/N_0 = 20.0dB, and L = 1.
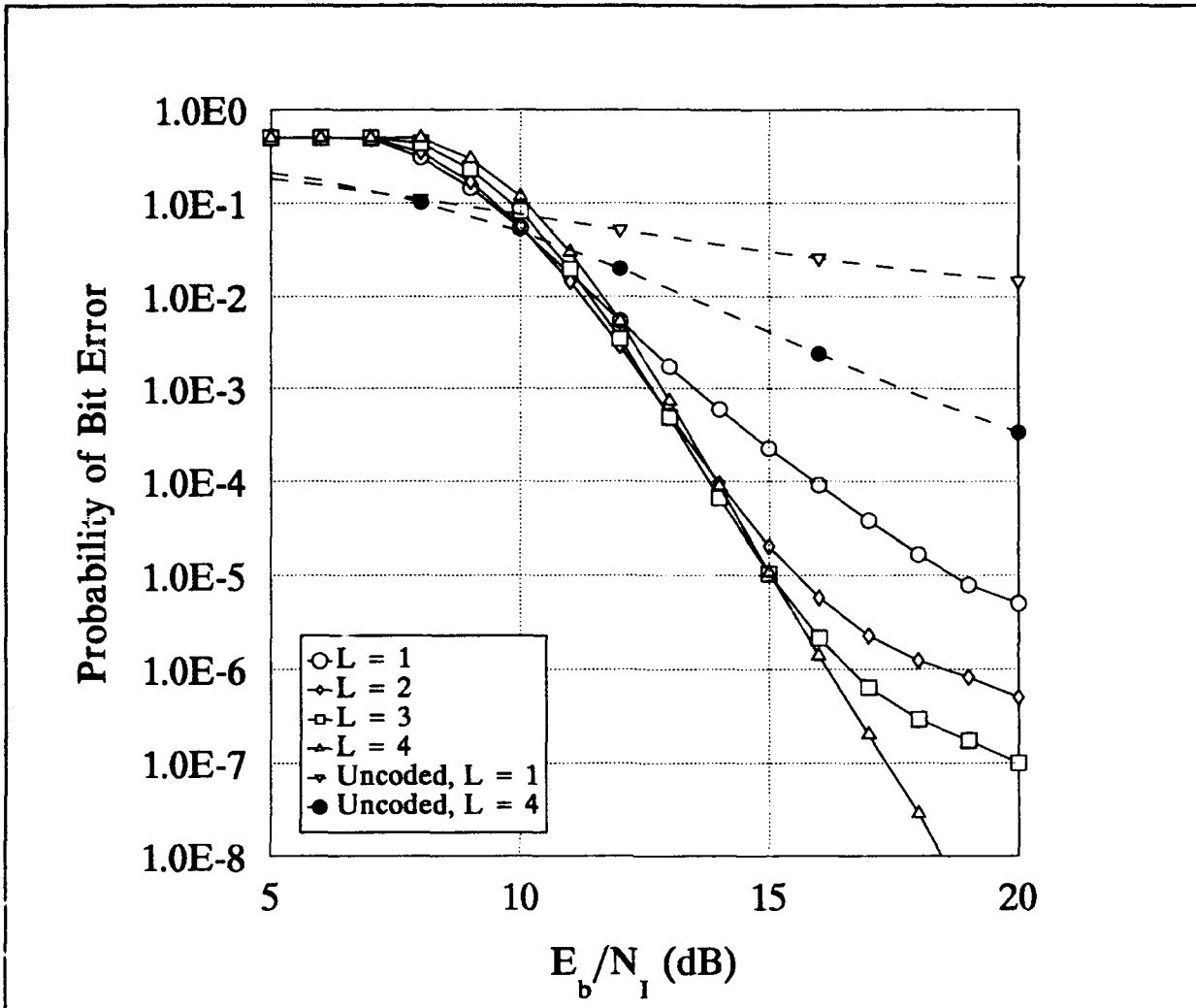
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Fig. 21 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decodi
for $\nu = 2$, 4 and 6 with Rician fading, a weak direct signal ($\alpha^2/2\sigma^2 = 1$), full band jammir
$E_b/N_0 = 20.0$dB, and L $\approx$ 2.

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Fig. 22 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for number of hops/bit of L = 1, 2, 3 and 4 with Rician fading, a weak direct signal ($\alpha^2/2\sigma^2$ = 1), full band jamming, E_b/N_0 = 20.0dB, and ν = 2.
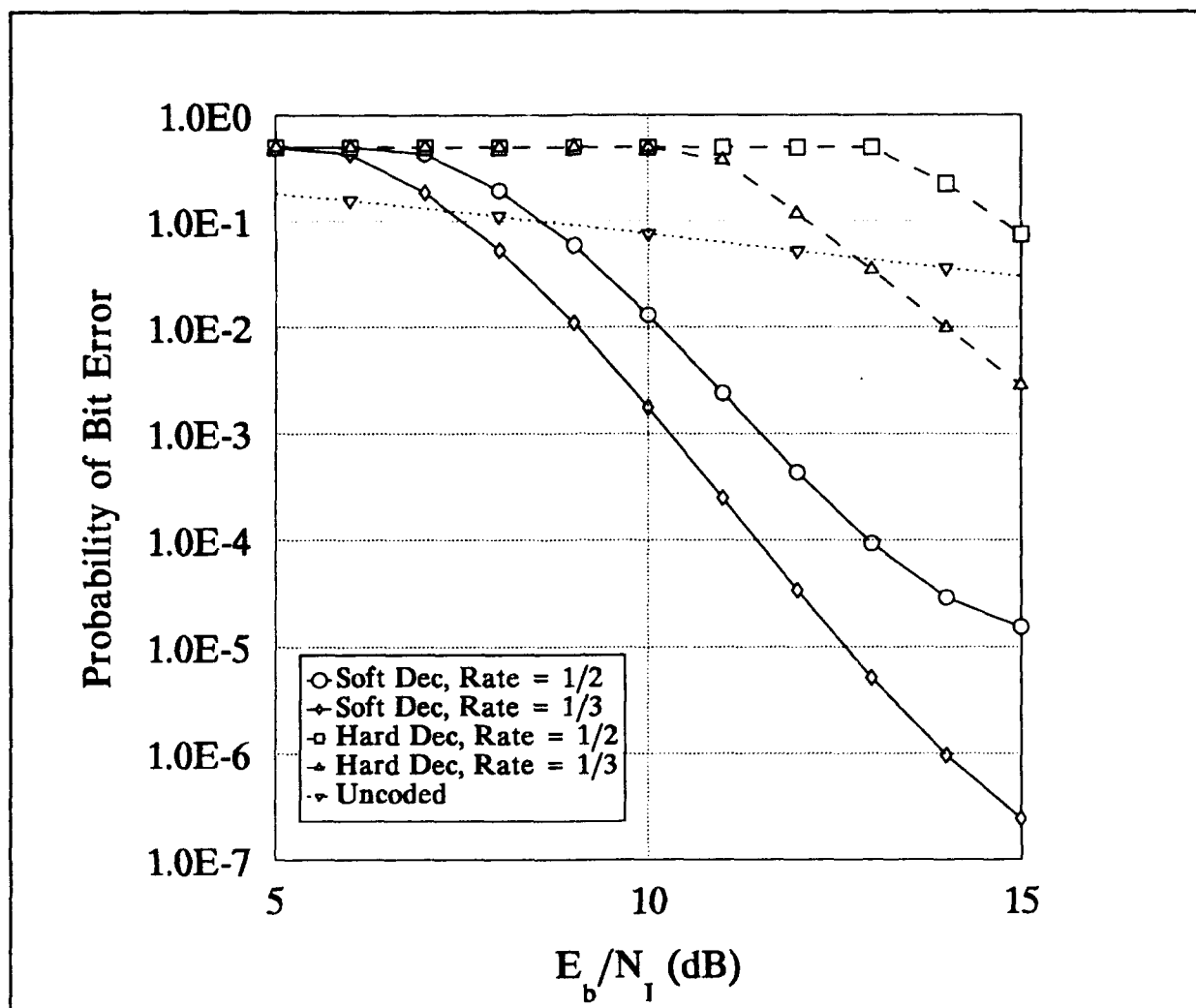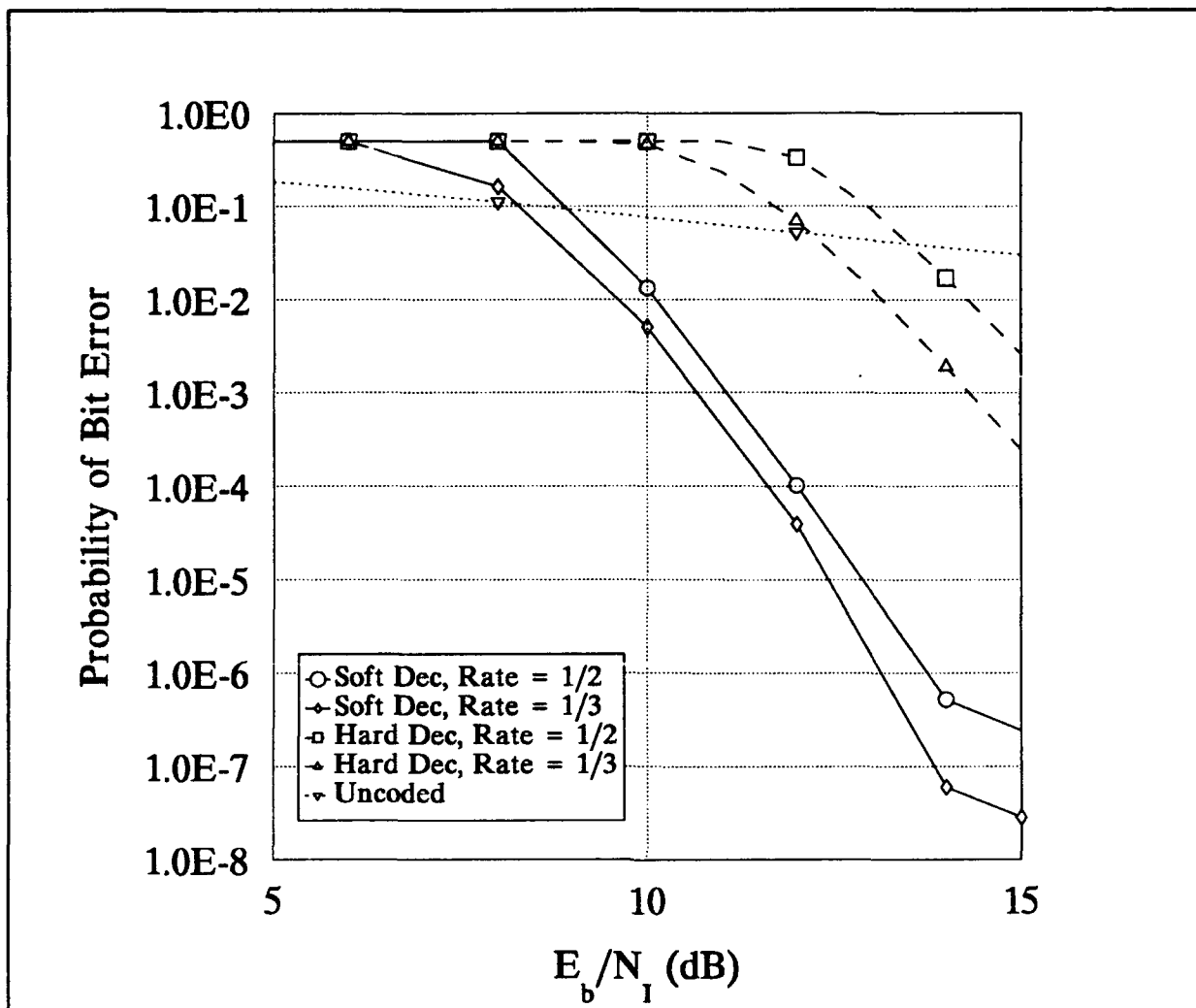
Fig. 23 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate, r = 1/2 and 1/3 with Rician fading, a weak direct signal ($\alpha^2/2\sigma^2 = 1$), full band jamming, $E_b/N_0 = 20.0$dB, L = 1, and $\nu = 6$.

Fig. 24 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate, r = 1/2 and 1/3 with Rician fading, a weak direct signal ($\alpha^2/2\sigma^2 = 1$), full band jamming, $E_b/N_O = 20.0$dB, L = 2, and $v = 6$.
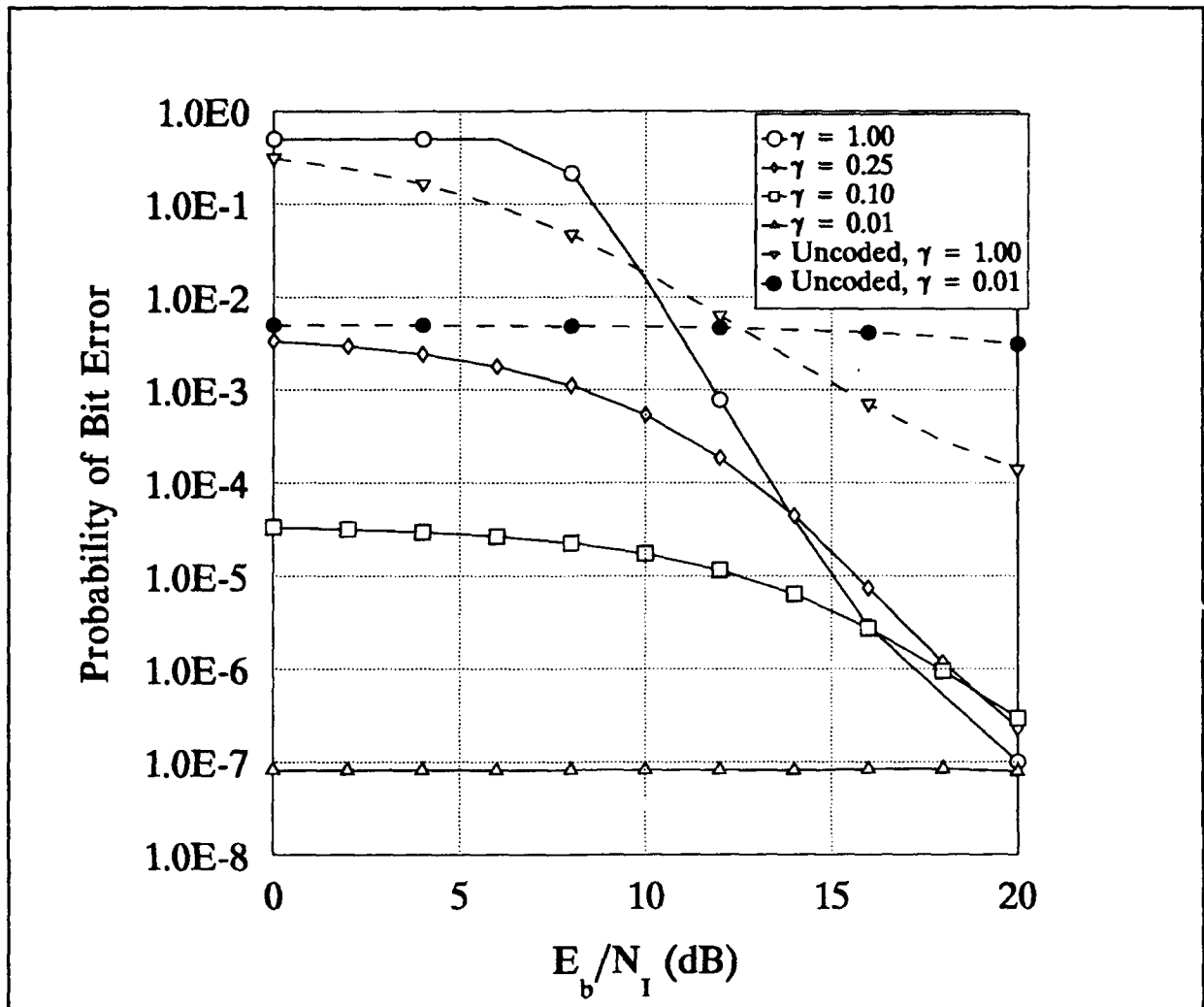
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Fig. 25 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for $\gamma = 1.0$, 0.25, 0.1 and 0.01 with Rician fading, a very strong direct signal ($\alpha^2/2\sigma^2 = 100$), $E_b/N_0 = 13.35$dB, L = 1, and $\nu = 2$.
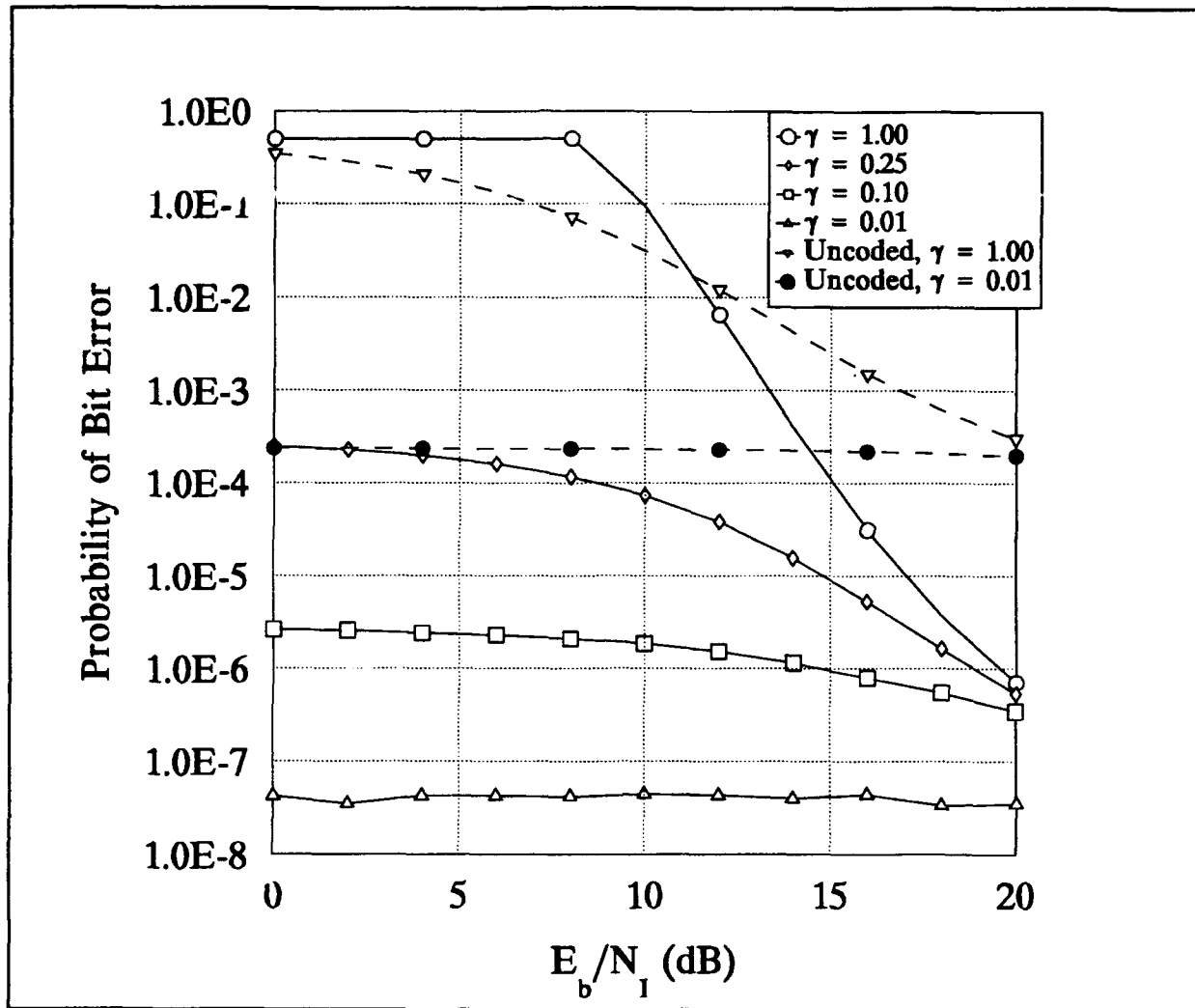
Fig. 26 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for $\gamma = 1.0$, 0.25, 0.1 and 0.01 with Rician fading, a very strong direct signal ($\alpha^2/2\sigma^2 = 100$), $E_b/N_0 = 13.35$dB, $L = 2$, and $v = 2$.

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